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Revisiting Louis de Broglie's famous 1924 paper in the *Philosophical Magazine*

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De Broglie's contribution in the *Philosophical Magazine* from 1924 is fascinating from many standpoints: for its moderate use of mathematics, the close connection to Einstein's special theory of relativity, and of course for the proposal of matter waves. We revisit this mostly speculative publication, which contributed crucially to the birth of quantum mechanics.

1. Introduction

The *Philosophical Magazine* is full of very famous papers that made science history and many that in essence gave birth to a completely new epoque of scientific thinking. One of these very crucial publications in the history of physics is that of de Broglie in 1924 [1]: this paper, reproduced as a facsimile at the end of this article, was the 'kick-off' of quantum mechanics. In order to make the historical paper readable for non-experts in the field I will recall as a kind of preliminary first some very basic concepts of physics in a hopefully simple and brief manner (and excuse myself to the experts for doing so).

1.1. *Einstein's special theory of relativity*

In 1905 Einstein surprised the physics community by stating that the tacit assumption of a constant mass m in Newton's second law,

$$F = \frac{d(mw)}{dt}, \tag{1}$$

w being the velocity†, is void, and that m has to be corrected in the following way

$$m = \frac{m_0}{\sqrt{1 - w^2/c^2}}, \tag{2}$$

where m_0, the so-called rest mass, is the mass of a body that is not moving and c is the speed of light.

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†In order to distinguish the velocity usually denoted by v, sufficiently well form the frequency, traditionally abbreviated by v, I take the liberty of denoting the (phase) belocity by w.

1.2 *Motion of waves*

The velocity (phase velocity) w [$\mathrm{cm\,s^{-1}}$] of propagating waves can simply be expressed in terms of the wavelength λ [cm] and the frequency ν [$\mathrm{s^{-1}}$]

$$w = \lambda\nu = \frac{\lambda}{T}, \tag{3}$$

where T is the period [s].

2. A 'very natural assumption'

Let us now first follow de Broglie's **very natural assumption**†: Let w be the velocity of a **light quantum** of frequency υ, where w is **nearly close to Einstein's limiting velocity** c, and let us assume that all such **light quanta** are of the same mass m_0. Then the energy W of one such quantum has to fulfil the relation

$$W = h\upsilon = \frac{m_0 c^2}{\sqrt{1-\beta^2}}, \quad \beta = \frac{w}{c}. \tag{4}$$

This, in essence, is de Broglie's very first argument not mentioning, however, at this stage what h refers to, but this equation seems **natural**, since according to Bohr's theory **atoms can only emit or adsorb radiant energy of frequency** υ **by finite amounts equal to** $h\upsilon$.

Equation (4) is of course in complete agreement with Einstein, but, when turned around,

$$\sqrt{1-\beta^2} = \frac{m_0 c^2}{h\nu}, \tag{5}$$

$$\beta = \sqrt{1 - \frac{m_0^2 c^4}{h^2\nu^2}}, \tag{6}$$

already poses a fundamental problem, namely of how to linearize a square root, i.e., a problem that was solved only years after by Dirac. However, since w is close to c, as de Broglie argues, he simply uses a binomial series, namely that for $(1-x)^{1/2}$, valid for $|x| \leq 1$, and by truncating the series after the first term he arrives at the expression

$$\beta = \frac{w}{c} = \sqrt{1 - \frac{m_0^2 c^4}{h^2\nu^2}} \simeq 1 - \frac{1}{2}\frac{m_0^2 c^4}{h^2\nu^2} \sim 1, \tag{7}$$

from which he concludes that m_0 has to be very small indeed, **at most of the order of** 10^{-50} [g]. The possibility that the velocity of all light quanta might equal

†All quotations from de Broglie's paper in the *Philosophical Magazine* are in bold.

Einstein's limiting velocity he carefully excluded, since then of course according to his argument the mass of **atoms of light** had to be zero,

$$\beta = 1, \quad \forall \upsilon \to m_0 = 0. \tag{8}$$

It is indeed interesting to see that starting from the by now famous identity

$$W = h\upsilon, \tag{9}$$

de Broglie arrives at a wild speculation about the mass of **light quanta**.

It would be extremely arrogant of us *a posteriori* to say but we do know that photons – as we call these light quanta now – have no mass, since in the early 1920s this was not clear at all. As all that was still an enigma, de Broglie tries in the following sections of his paper to hook up his **natural assumption** with some of – at that time – already well-known facts or assumptions.

3. Black radiation and 'Planck's hypothesis'

As in de Broglie's *very* **natural assumption** $w \sim c$, equation (4) can be written approximately as

$$W = \frac{m_0 c^2}{\sqrt{1-\beta^2}} \sim \frac{m_0 c w}{\sqrt{1-\beta^2}}, \tag{10}$$

which implies, by considering only the corresponding units, that W/c must be a momentum

$$G = \frac{W}{c} = \frac{m_0 w}{\sqrt{1-\beta^2}}, \tag{11}$$

namely the (modulus of the) momentum of one light quantum.

Now, if this indeed is a momentum, clearly the question arises as to what kind of distribution law, i.e., what kind of statistical mechanics, applies? Is it a classical one? Is Maxwell's energy partition law the basis of further understanding? Does one need to consider a gas of light quanta? Clearly enough, despite all attempts of Ernst Mach, Boltzmann statistics was already well accepted at de Broglie's time, but – as we now know – is only valid in the thermodynamical limit. At this point in his paper de Broglie recalls an earlier study [2] of his in which he showed that the phase space element $d\Omega$ to be used in statistical mechanics has to be scaled down by exactly the constant that Planck had suggested, namely by the quantity h, which implicitly was already used in equation (4),

$$d\Omega = \frac{1}{h^3} dx\, dy\, dz\, dp_x\, dp_y\, dp_z. \tag{12}$$

In that study he had arrived at the following density u_ν of the radiant energy

$$u_\nu d\nu = \frac{8\pi h}{c^3} \nu^3 e^{-h\nu/kT} d\nu. \tag{13}$$

With respect to the results in equations (12) and (13), de Broglie adds the following comment: **This was an encouraging result, but not quite complete. The assumption**

of finite elements of extension in phase space seemed to have a somewhat arbitrary and mysterious character. The result in equation (13) – as he states – was only possible by **using Planck's hypotheses** in equation (12).

At this stage we simply have to admire de Broglie's speculations, since by 'simply guessing' the form of equation (12) Heisenberg's uncertainty principle was already anticipated and of course the remaining mystery about the type of statistics to be applied (... **I was obliged to suppose some kind of quanta aggregation** ...) was only cleared up by the – to us now quite familiar – concepts of quantum statistics, i.e., by the Bose–Einstein statistics in the case of photons (**light quanta, light atoms**). But, and this is important to stress, the famous factor h^{-3} seemed to be reasonable already in 1922 (!).

Of course now, more than 80 years after, we might think that his expression for the pressure p on the wall of a *gas of **light quanta*** in the context of Einstein's theory of relativity appears a bit far fetched, since in plugging the momentum G from equation (11) into the well-known expression for the pressure (isotropical distribution of velocities) de Broglie arrives proudly at the expression

$$p = \frac{n}{6}(2Gc) = \frac{1}{3}nW, \tag{14}$$

where n is the number of light quanta in a volume element, which differs by a factor of two from the traditional (Maxwell–Boltzmann) formulation. However, the arguments leading to equation (14) illustrate perfectly the spell that Einstein's theory was exerting on the physics community in the early 1920s and seemingly partially still does, considering that in 2005 ('A hundred years of the special theory of relativity') the world-wide Year of Physics turned out to be a Year of Einstein.

4. Dynamics and Bohr's stability conditions

The third major type of argument de Broglie discusses is directed at the concept of time. From equation (4) it follows immediately that the frequency υ can be expressed in terms of the energy W,

$$\upsilon = \frac{W}{h} = \frac{1}{h}\frac{m_0 c^2}{\sqrt{1-\beta^2}}, \quad \beta = \frac{w}{c}, \tag{15}$$

which, multiplied by β^2, yields

$$\beta^2 \upsilon = \frac{w^2}{c^2}\upsilon = \frac{1}{h}\beta^2 \frac{m_0 c^2}{\sqrt{1-\beta^2}}. \tag{16}$$

In order to avoid the rather complicated arguments given by de Broglie, let us assume that on the left-hand side β is about 1,

$$\frac{w^2}{c^2}\frac{w}{\lambda} \sim \frac{w}{\lambda} = \frac{1}{h}\frac{m_0 \beta^2 c^2}{\sqrt{1-\beta^2}}, \tag{17}$$

and, furthermore, let us recall that w is the phase velocity,

$$\nu = \frac{1}{T} = \frac{w}{\lambda},$$

where λ is the wavelength and T is the period, see also equation (3). If ds denotes an element of the path of a wave moving from one crest to the next, then

$$\int \frac{ds}{\lambda} = n, \tag{18}$$

where n is the number of periods T [s], or the number of times this wave assumes the value of its amplitude. Integrating therefore the right-hand side of equation (17) over the time T has to yield the same result as the line integral on the left-hand side of this equation, i.e.,

$$\int \frac{ds}{\lambda} = \int_o^T \frac{1}{h} \frac{m_0 \beta^2 c^2}{\sqrt{1-\beta^2}} dt = n. \tag{19}$$

This is now, as de Broglie says, an **interesting explanation of Bohr' stability conditions**, which states that 'the motion (of an electron) can only be stable if the phase wave is tuned with the length of the path'. Equation (19) is indeed astonishing, since – as already said – on the left-hand side there is a line integral and on the right-hand side an integral over time, and also because de Broglie treats photons as if they were one of Bohr's electrons. With equation (19) de Broglie opened the door to 'wave mechanics' as it was called then, a door through which Schrödinger would be passing only two years later. In Schrödinger's first paper [3], which can be regarded as the actual starting point of quantum mechanics, he very honestly admits that all his thinking started from the witty ideas of de Broglie. Schrödinger gives complete reference to equation (19) as the origin of all his investigations by citing de Broglie's thesis [5], which in turn is the basis of the paper in the *Philosophical Magazine*.

5. Discussion and remarks

Rereading de Broglie's paper in the *Philosophical Magazine* makes clear that in 1924 quite a few aspects that nowadays seem to be completely familiar to us were a total enigma, as de Broglie admits. It is indeed remarkable that he occasionally speaks of **atoms of light** when he talks about photons. This in turn is a very old idea, namely that not only matter *per se* cannot be divided infinitely (Democritos), but also that time can only be chopped down to indivisible units; see, for example, [6] (*Zeitatome*), which to some extent corresponds to the 'phase-space volume factor' h^3, i.e., refers to a concept closely related to Heisenberg's uncertainty principle. de Broglie deals with photons and relates them to the orbits of an electron in Bohr's atomic model, since in 1924 a distinction between 'fermions' and 'bosons' did not exist nor was it thinkable! In principle, the main idea about this particular paper is to make use of **Einstein's correcting factor** β by cleverly equating this factor in the right place to unity.

There is of course one final observation to be made: in terms of the present politics of publishing scientific papers, de Broglie's contribution could never have been published because it only essentially contains speculations. However, one can just as well say that this paper proves that speculations are an essential part of physics; without them no new ideas and theories are born. Quantum mechanics has to be regarded as a true rupture in the history of physics, as a revolution in the philosophy of science – a revolution that desperately needed speculations and deviations beyond well-accepted ways of thinking. It pays off to come back occasionally to one of the famous papers in the *Philosophical Magazine*!

References

[1] L. de Broglie, Phil. Mag. **47** 446 (1924).

[2] L. de Broglie, J. de Physique (November 1922).

[3] E. Schrödinger, Ann. Phys. **79** 361 (1926).

[4] 'Vor allem möchte ich nicht unerwähnt lassen, daß ich die Anregung zu diesen Überlegungen in erster Linie den geistvollen Thesen des Hrn. Louis de Broglie verdanke und dem Nachdenken über die räumliche Verteilung von jener Phasenwellen, von denen er gezeigt hat, daß ihrer stets eine ganze Zahl, entlang der Bahn gemessen, auf jede Periode oder Quasiperiode des Elektrons fallen'. ('I would like to stress here that primarily the witty theses of Mr. L. de Broglie lead me to these ideas and to thinking about the spacial distribution of such phase waves, where he proved that always an integer number, measured along the orbit, falls on each period or quasi period of the electron').

[5] L. de Broglie, Ann. de Physique **3** 22 (1925)

[6] Moses Maimonides (1135–1204), *Guide to the Perplexed* (Dover, New York).