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COMMENTARY

Niels Bohr and the dawn of quantum theory

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Bohr's atomic model, one of the very few pieces of physics known to the general public, turned a hundred in 2013: a very good reason to revisit Bohr's original publications in the Philosophical Magazine, in which he introduced this model. It is indeed rewarding to (re-)discover what ideas and concepts stood behind it, to see not only 'orbits', but also 'rings' and 'flat ellipses' as electron trajectories at work, and, in particular, to admire Bohr's strong belief in the importance of Planck's law.

Keywords: atomic structure; electron energy states; electron theory

1. Introduction

Between 1890 and about 1910, physics experienced a complete change of paradigms: Rydberg's line spectra [1], the fascination of experiments with 'Röntgen's rays' [2], and of course Rutherford's radiation phenomena [3,4] (the birth of nuclear physics) had set the stage for a new age in science. Novel experiments recording, for example, the speed of light [5] or the elementary electric charge [6] provided up-to-then little-known natural constants with astonishing precision. Furthermore, the emission of particles [7–9] ('α-rays') clearly no longer could be described theoretically in terms of differential equations familiar from (classical) mechanics. In particular, Einstein's relativity theory [10] seemed to contradict accustomed schemes: the concept of mass *per se* (a cornerstone of Newtonian mechanics) had to be reconsidered in the view of its relation to the speed of light.

In Britain, Rutherford was claimed by many to be the greatest experimentalist since Faraday. Quite naturally, his lab at Manchester University quickly became a major center of attraction for young scientists. Among his foreign visitors, was, for example, a postdoc from Denmark by the name of Niels Bohr (1885–1962), who within only a few years became world famous. As is probably well known, Bohr's series of papers [11–16], all written between August 1912 and August 1915 and published in the Philosophical Magazine, are usually regarded as the very beginning of the 'quantum age'. Exactly a hundred years later, it seems most appropriate to revisit his original contributions to that journal.

In the following, the main ideas in his series of papers are recalled, giving particular attention to Part I [12] of the 'Trilogy' [12–14], see Figure 1, that contains already most of the concepts he proposed. In order to facilitate an easy reading, quotations from these

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I. *On the Constitution of Atoms and Molecules.*
By N. BOHR, *Dr. phil. Copenhagen* *.

Figure 1. Part I of Bohr's 'Trilogy' in the Philosophical Magazine [12].

papers are set in italics, Bohr's original equations are labelled according to their appearance in a particular part of the 'Trilogy' or by 'EMF', referring in turn to his attempts [15] to describe the influence of electric and magnetic fields on line spectra.

2. Collision theory

In his first contribution to the Philosophical Magazine [11], Bohr followed strictly the scientific vision of Rutherford, who, after all, provided a first realistic concept (model) of atoms that already included J.J. Thomson's discovery (1898) of 'electrons' (β- or cathode-rays):

> According to the theory given by Professor Rutherford of the scattering of α-rays by matter, the atoms of the matter are supposed to consist of a cluster of electrons kept together by attractive forces from a nucleus. This nucleus, which possesses a positive charge equal to the sum of the negative charges on the electrons, is further supposed to be the seat of the essential part of the mass of the atom, and to have dimensions which are exceedingly small compared with the dimensions of the atom. [11]

It seems that in the beginning, Bohr was fascinated by all the collision experiments performed in Manchester and by the success; this new type of physics made:

> In considering the collisions between the electrons and the particles, we shall at first neglect the forces from the side of the atoms. Let E and M be the charge and the mass of a particle, and e and m be corresponding quantities for an electron. Let us further assume that the electron is at rest and that the particle has a velocity V before the collision, and let the distance apart of the electron from the path of the particle before the collision be p ... . [11]

The velocity of the electron [v] after the collision will make an angle equal to $\frac{\pi}{2} - \vartheta$ with the path of the particle before the collision, and its value will be given by

$$v = V \frac{M}{M + m} 2 \sin \vartheta,$$

[where] 2ϑ is the angle through which the direction of the relative motion is deflected by the collision. [11]

It might sound strange now that he viewed the electron fixed in space and the colliding particle as the moving object. However, for *the rate of decrease of velocity of moving electrified particles as a function of the velocity of the particles and the number and frequencies of the electrons* [his main result, Equation (4) in [11]], he carefully gives a list of assumptions made, among which the condition:
(1) *That the velocity τ of the electrons in their undisturbed orbits is small compared with the velocity of the particles*[11]
shows, how little was known in 1913 about the nature of electrons.

It is interesting to note that Bohr started out in his first publication with collision theory, a scientific field that became extremely important later on, once the bombardment of uranium nuclei with neutrons was at hand. As is probably well known, his contributions to the theory of nuclear fission were and are subject of many books and articles, dealing, e.g. with the Manhattan project. Even a famous theatre play [17] winds around this topic.

3. The 'Trilogy'

Bohr's subsequent series of three papers from the same year (1913), entitled by *On the constitution of atoms and molecules* [12–14], very soon was termed 'The Trilogy' and quickly became known worldwide. Although his model of the electronic structure of an atom, nowadays, sometimes called 'old quantum theory', seems to be almost common knowledge, see Figure 2, it is worthwhile to go through all the important steps that let him to propose his concept. It is interesting to note that all three parts quote as address simply 'Copenhagen'.

3.1. *Trilogy, Part I*

In Part I [12] of his famous 'Trilogy', Bohr first repeats in the introduction, the definition of Rutherford's model for atoms by adding an interesting new bit of information about the number of electrons per atom:

> According to this theory [Rutherford's theory of an atom], the atoms consist of a positively charged nucleus surrounded by a system of electrons kept together by attractive forces from the nucleus; the total negative charge of the electrons is equal to the positive charge of the nucleus. Further, the nucleus is assumed to be the seat of the essential part of the mass of the atom, and to have linear dimensions exceedingly small compared with the linear dimensions of the whole atom. The number of electrons in an atom is deduced to be approximately equal to half the atomic weight. [12]

3.1.1. *Orbits*

In essence, the atomic model he then proposes in the 'Trilogy' is based on two important steps, the first of which, the 'classical' one, consists of an 'unorthodox' use of circular frequencies:
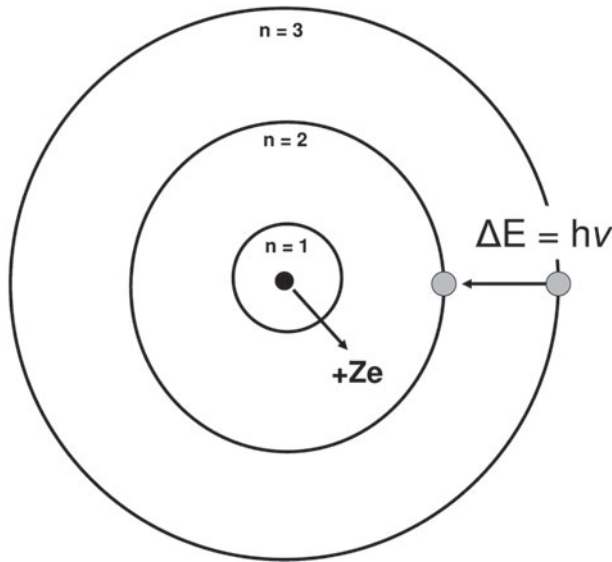
Figure 2. Bohr's atomic model as shown in innumerous (highschool) textbooks. Note that originally [12] Bohr used τ instead of n and N instead of Z. The image shown seems to belong to the generally accepted public knowledge of physics.

Let us at first assume that there is no energy radiation. In this case, the electron will describe stationary elliptical orbits. The frequency of revolution ω and the major-axis of the orbit $2a$ will depend on the amount of energy W which must be transferred to the system in order to remove the electron to an infinitely great distance apart from the nucleus. Denoting the charge of the electron and of the nucleus by $-e$ and E, respectively, and the mass of the electron by m, we thus get [12]

$$\omega = \frac{\sqrt{2}}{\pi}\frac{W^{3/2}}{eE\sqrt{m}}, \qquad 2a = \frac{eE}{W}. \qquad \text{(Trilogy I/1)}$$

Turning around his equation and using $eE = 2aW$,

$$W = \frac{m}{2}(2a\pi\omega)^2 = \frac{1}{2}mv^2,$$

it is easy to see that he simply proposed to replace the velocity v of a freely moving particle of mass m by a circular frequency corresponding to a circular orbit of radius a.

3.1.2. *Orbits and Planck's quantum of action*

The impact of Planck's theory of a quantum of action [18,19] on the physics community must have been enormous,

Whatever the alteration in the laws of motion of the electrons may be, it seems necessary to introduce in the laws in question a quantity foreign to the classical electrodynamics, i.e. Planck's constant, or as it often called the elementary quantum of action.[12]

since as the second important step, Bohr assumes that half of the emitted frequency ν in Planck's expression for the radiation energy, *$\tau h\nu$, where τ is an entire number, and h is a universal constant** [cited is [20]] can be related in his Equation (Trilogy I/1) to ω:
   Putting

$$W = \tau h \frac{\omega}{2}, \qquad \text{(Trilogy I/2)}$$

we get by help of the formula (1) [Equation (Trilogy I/1)] [12]

$$W = \frac{2\pi^2 m e^2 E^2}{\tau^2 h^2} \; , \; \omega = \frac{4\pi^2 m e^2 E^2}{\tau^3 h^3} \; , \; 2a = \frac{\tau^2 h^2}{2\pi^2 m e E}. \qquad \text{(Trilogy I/3)}$$

As Bohr points out, the result of this simple arithmetics is indeed astonishing:

Putting in the above expressions, $\tau = 1$ and $E = e$, and introducing the experimental values $e = 4.7.10^{-10}$, $e/m = 1.10^{17}$, $h = 6.5.10^{-27}$, we get $2a = 1.1.10^{-8}$ cm, $\omega = 6.2.10^{15}$ sec^{-1}, $W/e = 13$ volt. [12]
We see that these values are of the same order of magnitude as the linear dimensions of the atoms, the optical frequencies and the ionization-potentials.[12]

3.1.3. *Line spectra*

Even more astonishing is the application of Equation (Trilogy I/3) to the spectrum of hydrogen:

The amount of energy emitted by the passing of the system from a state corresponding to $\tau = \tau_1$ to one corresponding to $\tau = \tau_2$, is consequently [12]

$$W_{\tau_2} - W_{\tau_1} = \frac{2\pi^2 m e^4}{h^2} \left(\frac{1}{\tau_2^2} - \frac{1}{\tau_1^2} \right), \qquad \text{(Trilogy I/4)}$$

and accordingly, the frequency of the radiation is given by

$$\nu = \frac{2\pi^2 m e^4}{h^3} \left(\frac{1}{\tau_2^2} - \frac{1}{\tau_1^2} \right). \qquad \text{(Trilogy I/5)}$$

 At this stage, Bohr proudly remarks:

We see that this expression accounts for the law connecting the lines in the spectrum of hydrogen. If we put $\tau_2 = 2$ and let τ_1 vary, we get the ordinary Balmer series. If we put $\tau_2 = 3$, we get the series in the ultra-red observed by Paschen and previously suspected by Ritz. If we put $\tau_2 = 1$ and $\tau_2 = 4, 5, \ldots$, we get series, respectively, in the extreme ultraviolet and the extreme ultra-red, which are not observed, but the existence of which may be expected. [12]

   In order to appreciate the fascination that Bohr's first paper caused, one has to highlight the main, easy to understand assumptions, he made. Using his own words:

   (1) *That the dynamical equilibrium of the systems in the stationary states can be discussed by help of the ordinary mechanics, while the passing of the systems between different stationary states cannot be treated on that basis.*

(2) *That the latter process is followed by the emission of a homogeneous radiation, for which the relation between the frequency and the amount of energy emitted is the one given by Planck's theory.* [12]

Thus, the message was passed that it is sufficient to combine classical mechanics with Planck's radiation formula, an approach, which for a while achieved enormous popularity within the physics community. Perhaps, the only slightly unclear point was, why only half of the emitted frequency was to be related to Planck's equation. Although at a later stage, in his paper, Bohr tries to give an 'interpretation' for this factor $1/2$, the corresponding assumption remains, however, a bit mysterious, since clearly enough a (quantum mechanical) concept of probabilities was not yet available. Perhaps, it should be recalled that the constants in Equation (Trilogy I/5) multiplied by the square of the atomic number are usually called the **Ryberg constant** (R_H).

3.1.4. *Extensions to the He atom and molecules*

Because of the success in describing theoretically the line spectrum of Hydrogen, Bohr attempted to explain also more difficult cases such as the Helium atom and molecules. For He, he uses again his Equation (Trilogy I/3), which for $E = 2$ yields the following frequency ν of radiation

$$\nu = \frac{2\pi^2 m e^4}{h^3}\left(\frac{1}{(\tau_2/2)^2} - \frac{1}{(\tau_1/2)^2}\right), \qquad \text{(Trilogy I/6)}$$

and, as he notes, for $\tau_2 = 3, 4$ and varying τ_1 describes correctly the corresponding experimentally observed series of line spectra. Clearly for molecules, the situation turned out to be considerably more complicated: initially Bohr could only speculate and suggest an equation of the type

$$\nu = F_r(\tau_1) - F_s(\tau_2), \qquad \text{(Trilogy I/7)}$$

where the F_i are functions of τ.

3.1.5. *Important by-products*

In his attempts to give additional arguments for the validity of his theory, see Equations (Trilogy I/1)–(Trilogy I/3), he cites a simple analogon, which in the end gave rise to two famous constants:

> While there obviously can be no question of a mechanical foundation of the calculations given in this paper, it is, however, possible to give a very simple interpretation of the result …[first two relations in Equation (Trilogy I/3)] with the help of symbols taken from the ordinary mechanics. Denoting the angular momentum of the electron round the nucleus by M, we have immediately for a circular orbit $\pi M = T/\omega$, *where ω is the frequency of revolution and T the kinetic energy of the electron; for a circular orbit, we further have $T = W$ … , we consequently get*
>
> $$M = \tau M_0, \qquad \text{(Trilogy I/8)}$$
>
> where
>
> $$M_0 = \frac{h}{2\pi} = 1.04 \times 10^{-27}. \qquad \text{(Trilogy I/9)}$$

If we, therefore, assume that the orbit of the electron in the stationary states is circular, [then] … the angular momentum of the electron round the nucleus in a stationary state of the system is equal to an entire multiple of a universal value, independent of the charge on the nucleus.[12]

Clearly, one of the famous constants, derived by dividing Equation (Trilogy I/3) πW by ω, is $\hbar$ ($h/2\pi$) that became a standard quantity in quantum mechanics. Another famous constant connected with Bohr' name,

$$\mu_B = \frac{e\hbar}{2m_e}, \quad \text{(in SI units)}$$

where e is the elementary charge and m_e is the electron rest mass which was termed by Pauli a few years later (1920) **Bohr magneton**. Occasionally, this constant is also referred to as **Bohr-Procopiu magneton** [1].

3.1.6. *Permanent states of an atom*

In the last section of Part I, Bohr tries to support theoretically the model he proposed. Perhaps, this is the only section of this paper that viewed with present days' knowledge sounds completely outdated. However, one has to keep in mind that in 1913 very little was known about atoms. The only evidence available was a couple of experimental spectra and of course, Planck's law.

Considering systems in which more electrons are bound by a positive nucleus, a configuration of the electrons which presents itself as a permanent state is one in which the electrons are arranged in a ring round the nucleus.[12]
Let us consider a ring consisting of n electrons rotating round a nucleus of charge E, the electrons being arranged at equal angular intervals around the circumference of a circle of radius a [see Figure 3 for the case of $n = 4$].
The total potential energy of the system consisting of the electrons and the nucleus is

$$P = -\frac{ne}{a}(E - es_n), \tag{Trilogy 1/10}$$

where [12], [2]

$$s_n = \frac{1}{4}\sum_{s=1}^{s=n-1} \text{cosec}\frac{s\pi}{n}. \tag{Trilogy I/11}$$

For the radial force exerted on an electron by the nucleus and the other electrons, we get

$$F = -\frac{1}{n}\frac{dP}{da} = -\frac{e}{a^2}(E - es_n). \tag{Trilogy I/12}$$

Clearly, using simply the virial theorem, the (total) kinetic energy T is given by

$$T = \frac{ne}{a}(E - es_n) = nT_n, \quad T_n = \frac{e}{a}(E - es_n),$$

where

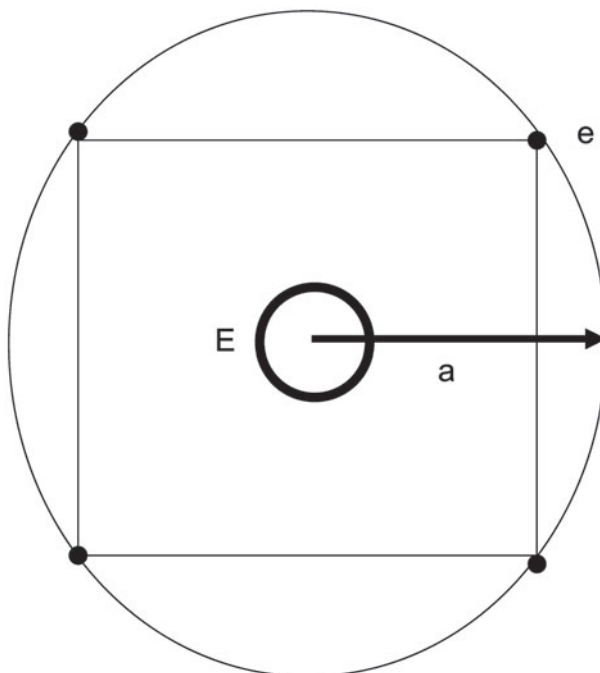$$T_n = \frac{e}{a}(E - es_n), \tag{Trilogy I/13}$$

Figure 3. Four electrons (e) on a circular ring, see Equation (Trilogy I/11). In the notation used by Bohr in Equation (Trilogy I/1) E is the nuclear charge and a the radius of the 'orbit'.

And, therefore, according to his arguments

$$T_n = nh\omega/2,$$

and the angular moment per electron, $M_0 = h/2\pi$, see Equations (Trilogy I/2) and (Trilogy I/9). Obviously, the arguments he uses to discuss *permanent states of an atomic system* are based on simple electrostatics and his conviction that Planck's quantum of action had to be part of any theory of atoms. In this sense, also the hypothesis expressed as a kind of summary at the very end of Part I:

> In any molecular system consisting of positive nuclei and electrons, in which the nuclei are at rest relative to each other and the electrons move in circular orbits, the angular momentum of every electron round the centre of its orbit will in the permanent state of the system be equal to $h/2\pi$ where h is Planck's constant. [12]

has to be understood, namely as a kind of hope *to throw some further light on the foundations* of this hypothesis in the following parts of his 'Trilogy' from *another point of view*.

3.2. *Trilogy, Part II*

In Part II of the 'Trilogy', Bohr immediately comes back to his hypothesis:

> We shall assume that the electrons are arranged at equal angular intervals in coaxial rings rotating round the nucleus. In order to determine the frequency and dimensions of the rings, we shall use the main hypothesis of the first paper, viz.: that in the permanent state of an atom,

the angular momentum of every electron round the centre of its orbit is equal to the universal value $h/2\pi$, where h is Planck's constant.[13]

in order to elaborate to quite some extent, again the stability of his coaxial rings. For this purpose, he slightly generalizes the arguments already given. Instead of using the simple expression for the radial force F in Equation (Trilogy I/12), he now states a condition of *dynamical equilibrium*

$$\frac{mv}{a} = \frac{e^2}{a^2} F, \qquad \text{(Trilogy II/1)}$$

where m and e are again the mass and charge of an electron, v its velocity and a is the radius of the ring. Together with what he calls *the universal constancy of the angular momentum of the electron*,

$$mva = \frac{h}{2\pi}, \qquad \text{(Trilogy II/2)}$$

this yields

$$a = \frac{h^2}{4\pi^2 e^2 m} F^{-1}, \quad v = \frac{2\pi e^2}{h} F, \qquad \text{(Trilogy II/3)}$$

and

$$\omega = \frac{4\pi^2 e^4 m}{h^3} F^2 \qquad \text{(Trilogy II/4)}$$

for the circular frequency (*frequency of revolution*). Consequently, *If F is known, the dimensions and frequency of the corresponding orbit are simply determined by Equations (Trilogy II/1) and (Trilogy II/2)*.[13]

Bohr's cautious remark that if the velocity v is not small when compared to the velocity of light, m in Equation (Trilogy II/2) has to be replaced by $m/\sqrt{1 - v^2/c^2}$, shows that Einstein's ideas (1905) about special relativity [10] had already penetrated quite a few fields in physics.

A substantial portion of Part II is devoted to discuss the stability of his rings by taking into account a more elaborate electrostatic model for the radial force or by allowing for perpendicular displacements of the position of the plane of electrons or of a ring. For example, as an illustration for the ansatz in Equations (Trilogy 1/10) and (Trilogy I/11), he actually lists a table of the s_n, see Equation (Trilogy I/11), and the increase of potential energy with respect to $n = 1$ for $n \leq 16$.

For the case of systems with $n \leq 4$, i.e. the H, He, Li and Be atoms, Bohr then uses Equation (Trilogy I/6) from Part I for the frequency ν

$$\nu = \frac{2\pi^2 m e^4}{h^3} \left(\frac{1}{(\tau_2/n)^2} - \frac{1}{(\tau_1/n)^2} \right),$$

in combination with Equations (Trilogy I/2) and (Trilogy I/3) to evaluate explicitly the respective radii a, circular frequencies ω and 'ionization potentials' W. Very typically for his approach, he relates all calculated quantities to the ground state of Hydrogen, a_0, ω_0

and W_0. Radii are, therefore, expressed in terms of multiples of a_0,

$$a_0 = \frac{h^2}{4\pi^2 e^2 m},$$

the famous **Bohr radius**, which still serves as measure of length when dealing with atomic dimensions.

3.3. *Trilogy, Part III*

In Maxwell's kinematic gas theory, as well as in Boltzmann's kinematic statistical theory, no distinction was made between atoms and molecules: both were treated as particles of some mass m. Almost like a sign of the new age in physics that had set in, the initial paragraph of Part III contains a surprisingly 'modern' definition of molecules:

> According to Rutherford's theory of the structure of atoms, the difference between an atom of an element and a molecule of a chemical combination is that the first consists of a cluster of electrons surrounding a single positive nucleus of exceedingly small dimensions and of a mass great in comparison with that of the electrons, while the latter contains at least two nuclei at distances from each other comparable with the distances apart of the electrons in the surrounding cluster.[14]

Self-critically, Bohr notes that according to his definition molecules did not really fit into the scheme introduced by him in Parts I and II:

> The leading idea used in the former papers was that the atoms were termed through the successive binding by the nucleus of a number of electrons initially nearly at rest. Such a conception, however, cannot be utilized in considering the formation of a system containing more than a single nucleus; for in the latter case, there will be nothing to keep the nuclei together during the binding of the electrons …. We must, therefore, assume that configurations containing several nuclei are formed by the interaction of systems …, which already have bound a number of electrons. …

> We shall consider only the simple case of a system consisting of two nuclei and of a ring of electrons rotating round the line connecting them [see Figure 4] … . As in the former papers, we shall assume that the conditions of equilibrium can be deduced with the help of the ordinary mechanics.[14]
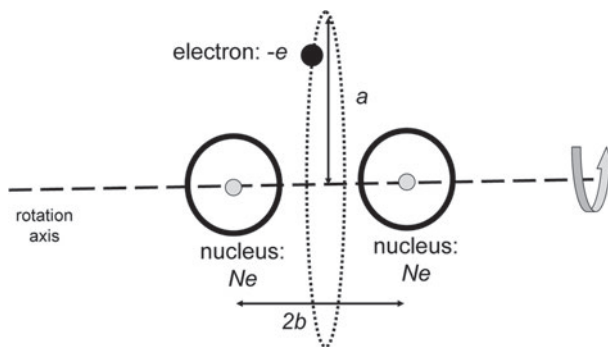

Figure 4. Bohr's concept of a diatomic molecule as expressed in [14]. Ne is the nuclear charge, $2b$ the interatomic distance and a the radius of the ring, see Equation (Trilogy III/1).

In order to explain what model he had in mind, Bohr considers a homonuclear diatomic molecule with nuclear charges Ne and n electrons, see Figure 4, and claims that the system is in equilibrium

if the nuclei are the same distance apart from the plane of the ring and if the ratio between the diameter of the ring 2a and the distance apart of the nuclei 2b is given by

$$b = a\left(\left(\frac{4n}{N}\right)^{2/3} - 1\right)^{-1/2}, \qquad \text{(Trilogy III/1)}$$

provided that the frequency of revolution ω is of a magnitude such that for each of the electrons, the centrifugal force balances the radial force due to the attraction of the nuclei and the repulsion of the other electrons.[14]

Denoting this centrifugal force by $\left(e^2/a^2\right)F$, then in terms of Equations (Trilogy II/1) and (Trilogy II/2)

$$a = \frac{h^2}{4\pi^2 e^2 m}F^{-1} \quad , \quad \omega = \frac{4\pi^2 e^2 m}{h},$$

where, as should be recalled, m is the mass of an electron and h is the Planck's constant. For a system with n electrons, F is then given by

$$F = \frac{N^2}{2n}\left(\left(\frac{4n}{N}\right)^{2/3} - 1\right)^{3/2} - s_n, \qquad \text{(Trilogy III/2)}$$

where (now)

$$s_n = \sum_{s=1}^{s=n-1} \operatorname{cosec}\frac{s\pi}{n},$$

see also Equation (Trilogy I/11) and Figure 3.

Quite obviously Bohr tried very hard to relate molecules to his most successful theory of atoms. He even discussed the displacement of the position of the plane in which the ring of electrons is situated and argues that in the case of diatomic molecules with a large number of electrons, the electrons would move in an increasing number of rings (orbits):

The [outer] ring, which keeps the system together, represents the chemical bond. [14]

For the H_2 molecule discussed, Bohr arrives at

$$b = \frac{1}{\sqrt{3}}a \quad , \quad F = \frac{3\sqrt{3} - 1}{4}, \qquad \text{(Trilogy III/3)}$$

which in terms of the corresponding quantities for a single H atom yields

$$a = 0.95a_o \quad , \quad \omega = 1.10\omega_0 \quad , \quad W = 2.20W_0,$$

where W is the total kinetic energy of the system. *Since $W > 2W_0$, it follows that two hydrogen atoms combine into a molecule with emission of energy.* [14]

It turned out that the numerical value of the formation energy of one mole of H_2, $(W - 2W_0)N_L$, where N_L is the Avogadro constant, namely $6.0.10^4$ cal, was only of the *right order of magnitude* when compared to the then existing experimental values.

Quite clearly viewed in terms of modern (quantum mechanical) theories of molecules, Bohr's model for molecules seems to be almost bizzare. However, one has to acknowledge

that the success of his model for atoms necessarily suggested to propose a similar concept for molecules. And of course, one has to remember again that nothing else was available, in particular, considering his definition of molecules.

4. Electric and magnetic fields

In his 1914 paper [15], Bohr tried to extend his theory of line spectra to the then only recently discovered Stark effect (1913) [21] and to the Zeeman effect (1897) [22], i.e. to include the presence of an electric or a magnetic field into the model discussed by him in his trilogy.

4.1. *The effect of an electric field*

With respect to the Stark effect, Bohr first cautiously considers the effects that possibly could cause this new phenomenon:

(1) *The field may influence the stationary states of the emitting system, and thereby, the energy possessed by the system in these states.*
(2) *It may influence the mechanism of transition between the stationary states, and thereby, the relation between the frequency of the radiation and the amount of energy emitted.* [15]

and only then argues that in the case of hydrogen, *through the effect of an external electric field, the orbit will be deformed. If the* [electric] *force is not accurately perpendicular to the plane of the orbit, …the orbit may at every moment be considered as an ellipse with the nucleus in the focus ….*[15]

In particular, he claims *that the problem only allows two stationary orbits of the electron. In these, the eccentricity* [3] *is equal to 1 and the major-axis parallel to the axis of the external field; the orbits simply consist of a straight line through the nucleus parallel to the axis of the field, one on each side of it,*[15] such that *by neglecting quantities proportional to the square of the magnitude of the external electric force* for $b \sim 0$, see Figure 5,
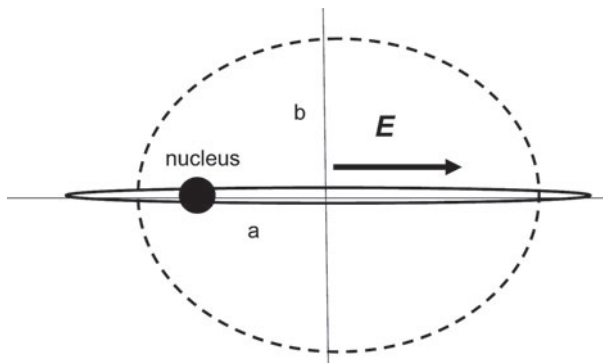

Figure 5. Bohr's orbits in the case of the Stark effect [15]. *a* and *b* are the major and minor semi-axis of an ellipses, *E* the external electric force. Stable states refer to $b \sim 0$.

$$\omega^2 = \frac{e^2}{4\pi^2 ma^3} \left(1 \mp 3E\frac{a^2}{e} \right), \tag{EMF-1}$$

$$T = \frac{e^2}{2a} \left(1 \mp 2E\frac{a^2}{e} \right), \tag{EMF-2}$$

where ω is the frequency of the *vibration*, E the external electric force and T the mean value of the kinetic energy of the electron during the vibration.

Since T has to be of the form $T = nh\omega/2$, see Equation (Trilogy I/13), *in the presence of an electric field, we shall, therefore, assume the existence of two series of stationary states of the hydrogen atom … . [Assuming] that the system can pass only between the different states in each series … we get for the frequency [ν] of the radiation emitted by a transition between two states corresponding to n_2 and n_1, respectively:*

$$\nu = \frac{2\pi^2 e^4 m}{h^3} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \left(1 \mp E\frac{3h^4}{16\pi^4 e^5 m^2} n_1^2 n_2^2 \right). \tag{EMF-3}$$

This formula gives for every hydrogen line two components situated symmetrically with regard to the original line. Their difference in frequency is proportional to the electric force

$$\Delta\nu = \frac{3}{4\pi^2} \frac{h}{em} E \left(n_2^2 - n_1^2 \right). \tag{EMF-4}$$

In trying to reproduce the experimental results for hydrogen Bohr remarks that using the experimental values for e, m, h and E, his results are of the same order of magnitude as those of Stark. He carefully adds, however, that

> The problem of the influence of an electric field on the spectra of other elements is naturally far more complicated than for hydrogen and cannot be discussed in detail until the theory for such spectra is further developed. It seems, however, possible on the present theory to obtain a simple explanation of [the data] observed by Stark.

4.2. *The effect of a magnetic field*

With respect to the Zeeman effect [22], Bohr had to admit that

> since in the presence of a magnetic field the spectrum of an element cannot be expressed by a formula of the type (2) [$\nu = f_r(n_1) - f_s(n_2)$, see also Equation (Trilogy I/7)], it follows that the effect of the field cannot be explained by considerations analogous to those employed … in considering the effect of an electric field.

He honestly also stated that he could not give any explanations for the then already available measurements that eventually led to the Paschen-Back effect and for the problem with double lines.

5. Quantum theory

In his last paper entitled 'On the Quantum Theory of Radiation and the Structure of the Atom' [16], Bohr tried to respond to the critique his theory had met and to make contact with the latest experimental evidence. For this purpose, he repeats and discusses most of the equations already introduced earlier, some of them in a slightly more compact form. The, perhaps, most interesting part of this publication is the series of lemmas, which not

only served as background for his earlier communications to the Philosophical Magazine, but to some extent, became also the 'bible' of the 'old quantum theory':

(A) *An atomic system possesses a number of states in which no emission of energy radiation takes place, even if the particles are in motion relative to each other, and such an emission is to be expected on ordinary electrodynamics. The states are denoted as the 'stationary' states of the system under consideration.*

(B) *Any emission or absorption of energy radiation will correspond to the transition between two stationary states. The radiation emitted during such a transition is homogeneous, and the frequency ν is determined by the relation*

$$h\nu = A_1 - A_2,$$

where h is Planck's constant and A_1 and A_2 are the energies of the system in the two stationary states.

(C) *That the dynamical equilibrium of the systems in the stationary states is governed by the ordinary laws of mechanics, while these laws do not hold for the transition from one state to another.*

(D) *That the various possible stationary states of a system consisting of an electron rotating round a positive nucleus are determined by the relation*

$$T = \frac{1}{2}nh\omega,$$

where T is the mean value of the kinetic energy of the system, ω is the frequency of rotation and n is a whole number.

(E) *In any atomic or molecular system consisting of positive nuclei and electrons in which the nuclei are at rest relative to each other, and the electrons move in circular orbits, the angular momentum of each electron round the centre of its orbit will be equal to $h/2\pi$ in the 'normal' state of the system, i.e. the state in which the total energy is a minimum.*

(F) *A configuration satisfying the condition E is stable if the total energy of the system is less than in any neighbouring configuration satisfying the same condition of angular momentum of the electrons.* Clearly, his warning, it must be emphasized that only in the case of circular orbits has the angular momentum any connexion with the principles of the Quantum theory, remains timeless in view of Schrödinger's papers [24].

6. Conclusion

Quite often the nineteenth century is regarded to end politically with the beginning of the First World War, and to be followed by a rather short twentieth century (terminated essentially by the fall of the Iron Curtain). Concurrently with the close of a long century, the years 1913–1915 have to be regarded as the dawn of quantum theory, which from then on would govern the concepts of modern physics (natural sciences).

The enormous success of Bohr's papers is partially based on the fact that he used rather simple mathematics, easily understandable formal arguments and simple models, see Figures 3–5. Furthermore, all theoretical concepts were very well explained in words. Clearly, also the comparison between corresponding experimental values and his numerical

results were immediately convincing. Within a few years, Bohr became world famous. Even today his atomic model can be found in most high-school physics textbooks.

Looking back, it seems that from his very first paper on till his contributions to nuclear fission, Bohr tried (with increasing knowledge) to answer Rutherford's (eternal) question of what is an atom.

As is probably well known, about 10 years after Bohr's fulminate series of papers in the Philosophical Magazine, de Broglie [23] attempted to make contact with Bohr's ideas, starting, however, from Einstein's theory of special relativity (in de Broglie words: *The present theory suggests an interesting explanation of Bohr's stability conditions* [23]). De Broglie's publication, which, e.g. served as incentive for Schrödinger's famous papers [24], initiated in the following years the area of quantum mechanics with its paradigmatic change from certainty to uncertainty.

Notes

1. Ştefan_Procopiu. http://en.wikipedia.org/wiki/Ştefan_Procopiu.
2. $\mathrm{cosec}(x) = \csc(x) = \sin^{-1}(x)$
3. The eccentricity e is defined as $\left(\left(a^2 - b^2 \right) / a^2 \right)^{1/2}$, where a and b is the major and minor semi-axis, respectively, see also Figure 5.

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[21] for an interesting illustration of the Stark effect as described by a fully relativistic scheme in the case of an iron atom see A. Vernes, P. Weinberger, and L. Szunyogh, Phys. Rev. B78 (2008) p.155129.

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