

XXXV. *A Tentative Theory of Light Quanta.*  
By LOUIS DE BROGLIE<sup>\*</sup>.

I. *The Light Quantum.*

THE experimental evidence accumulated in recent years seems to be quite conclusive in favour of the actual reality of light quanta. The photoelectric effect, which is the chief mechanism of energy exchange between radiation and matter, seems with increasing probability to be always governed by Einstein's photoelectric law. Experiments on the photographic actions, the recent results of A. H. Compton on the change in wave-length of scattered X-rays, would be very difficult to explain without using the notion of the light quantum. On the theoretical side, Bohr's theory, which is supported by so many experimental proofs, is based on the postulate that atoms can only emit or absorb radiant energy of frequency  $\nu$  which is equal to  $h\nu$ , and Einstein's theory of energy fluctuations in the black radiation leads us necessarily to the same ideas.

**Louis de Broglie  
and his time**

Do you remember the year 1999?

It's only 8 years ago that we talked about the upcoming millenium, about the time-devices in computers that would not make it!

De Broglies article was first published in 1922 – eight years after the First World War started



Max Planck

$$E = h \cdot \nu$$

### An die Kulturwelt! Ein Aufruf

Der nachstehende Aufruf ging uns zur Veröffentlichung zu: Wir als Vertreter deutscher Wissenschaft und Kultur erheben vor der gesamten Kulturwelt Protest gegen die Lügen und Verleumdungen, mit denen unsere Feinde Deutschlands reine Sache in dem ihm aufgezwungenen schweren Daseinskampfe zu beschmutzen trachten.

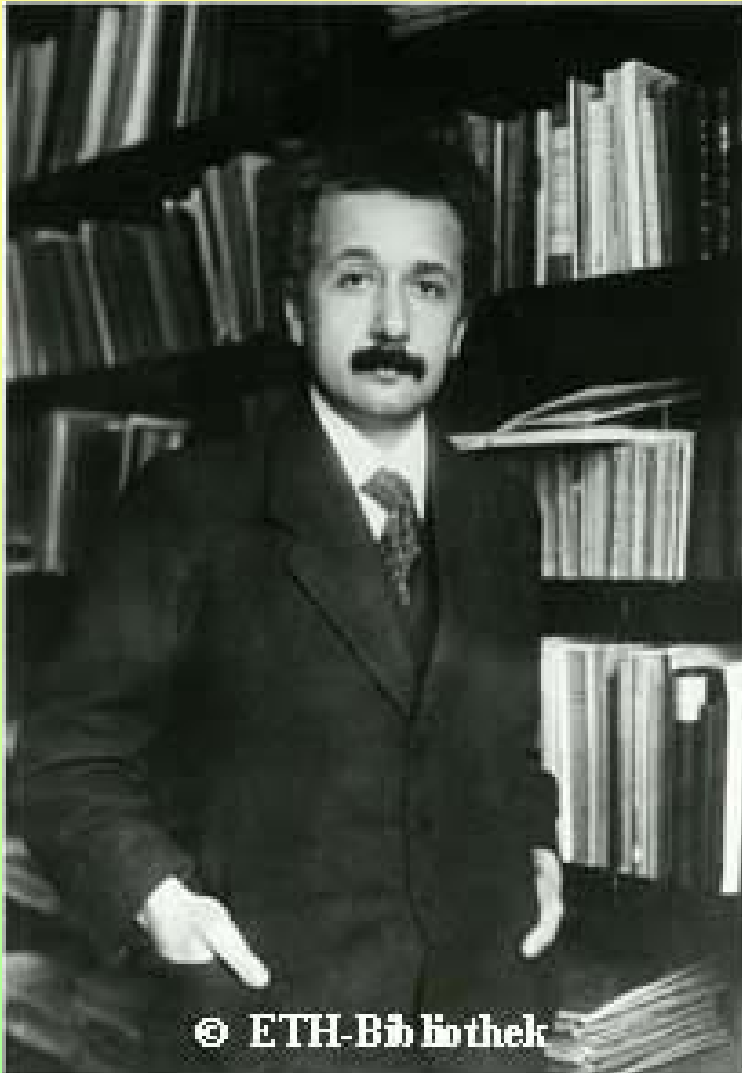
Glaubt uns! Glaubt, daß wir diesen Kampf zu Ende kämpfen werden als ein Kulturvolk, dem das Vermächtnis eines Goethe, eines Beethoven, eines Kant ebenso heilig ist wie sein Herd und seine Scholle.

Dafür stehen wir Euch ein mit unserem Namen und mit unserer Ehre! “

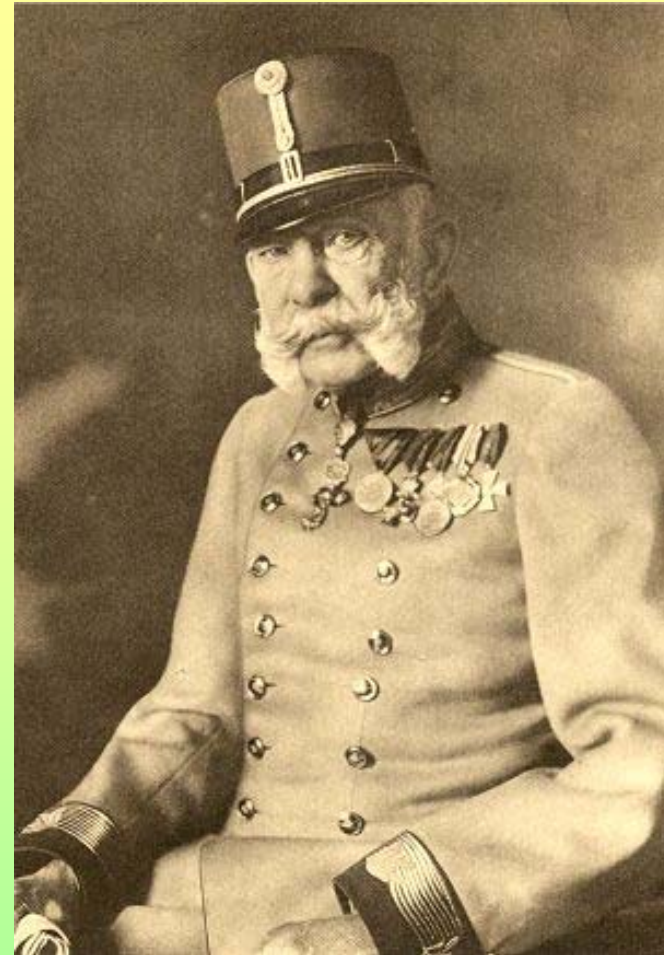
– Manifest der 93: (93 Unterzeichner –  
Berlin, 4. Oktober 1914)

u.a.: M. Planck

1916



Albert Einstein,  
im Kaiser-Wilhelm-Institut



Extraausgabe der "Kaiserlich Wiener Zeitung" meldet:  
Seine k. und k. Apostolische Majestät Kaiser Franz Josef I. sind heute, 21. November, 9 Uhr abends, im Schloß Schönbrunn sanft im Herrn entschlafen.



1916: Schlacht bei Verdun





**British 55th Division gas casualties 10 April 1918**



Otto Dix: Im Kampfgraben  
1916



Otto Dix: Volltreffer  
1918



**11. November 1918:  
Verzicht Kaiser Karls auf  
Anteil an den Staatsgeschäften**



Die Flucht des Kaisers,

**1919**



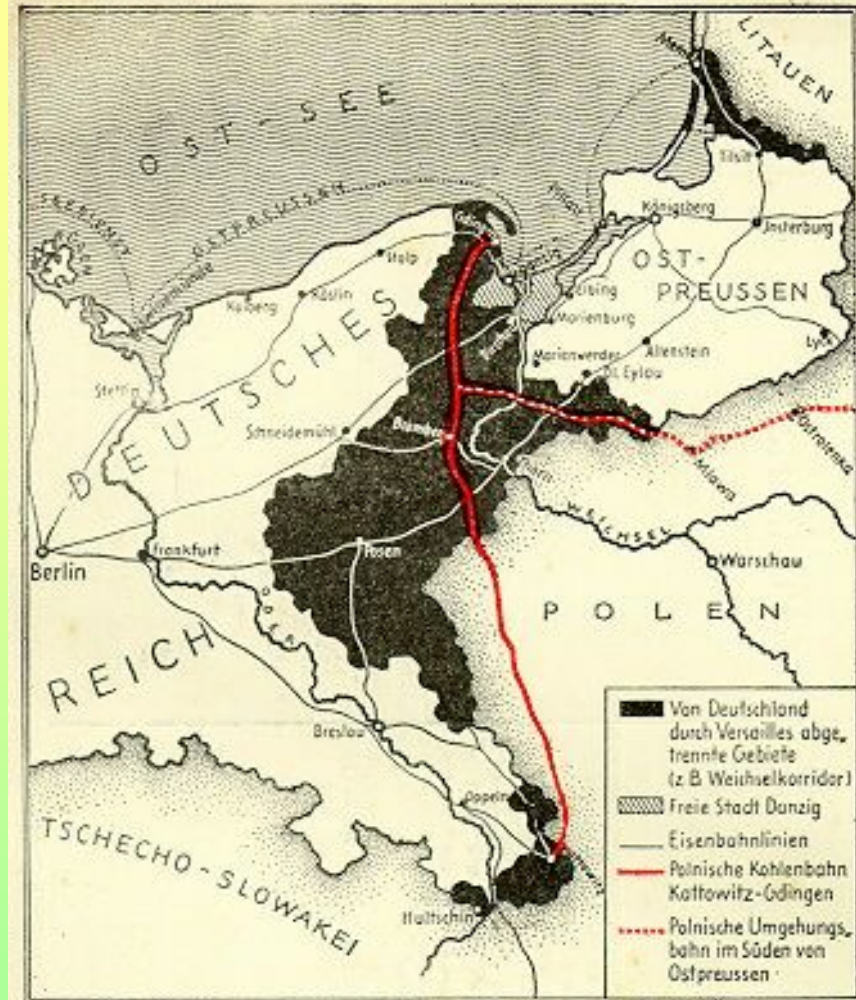
## Der deutsche Osten

Postkarte mit Gebietsabtretungen

des Deutschen Reichs

14,9 x 10,4 cm

1920



### Der deutsche Osten

wird charakterisiert durch das Ueberwiegen der Landwirtschaft, damit zusammenhängende dünne Bevölkerung, große Entfernungen, Ost — West — Verkehrslinien. Holz, Kartoffeln, Schweine und Getreide sind die wichtigsten Produkte. Industrie befindet sich nur im Süden — Oberschlesien und Waldenburger Bergland —, in den Städten des mittleren und nördlichen Ostens, z. B. Bromberg, Thorn, Graudenz, Pr. Stargard, Danzig, Königsberg. Die wirtschaftliche, kulturelle und politische Einheit deutschen Ostgebietes ist durch das Versailler Diktat zerschlagen. Das Hultschiner Ländchen, das oberschlesische Industriegebiet, Posen, Westpreußen und Memelland sind zurzeit an früher feindliche Staaten gefallen, Danzig zu einem sogenannten freien Staat geworden.

# Weimarer Republik



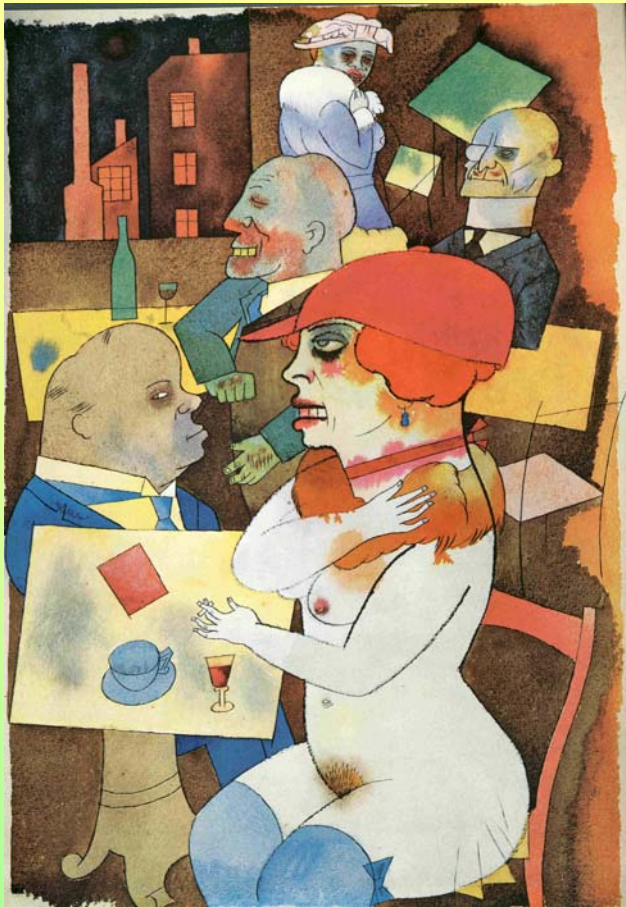
Käthe Kollwitz: Ermordung Karl Liebknechts, 1920



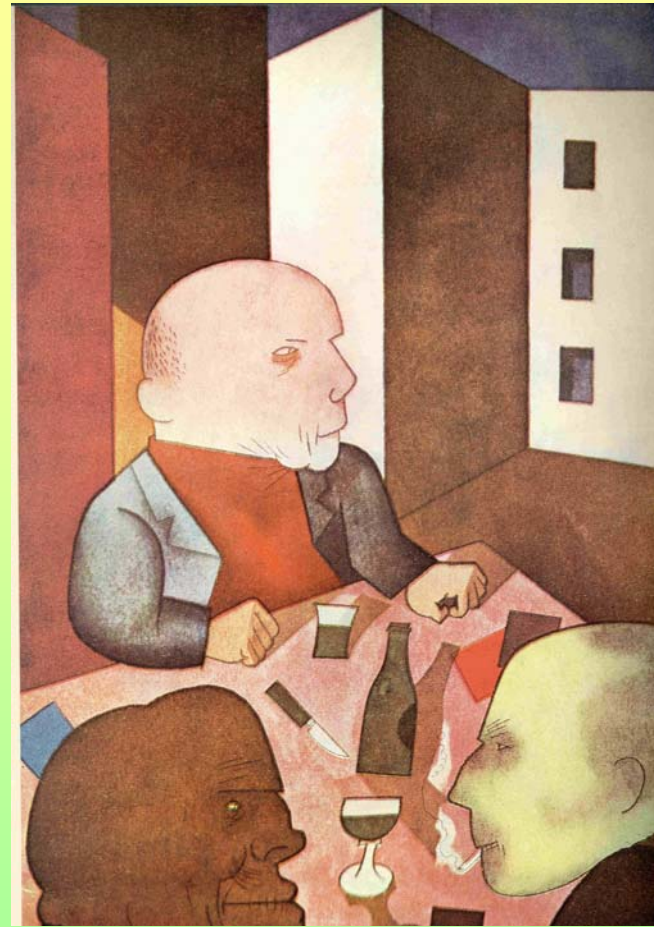
Otto Dix: Salon I, 1920



George Grosz: Ein grauer Tag,  
1921



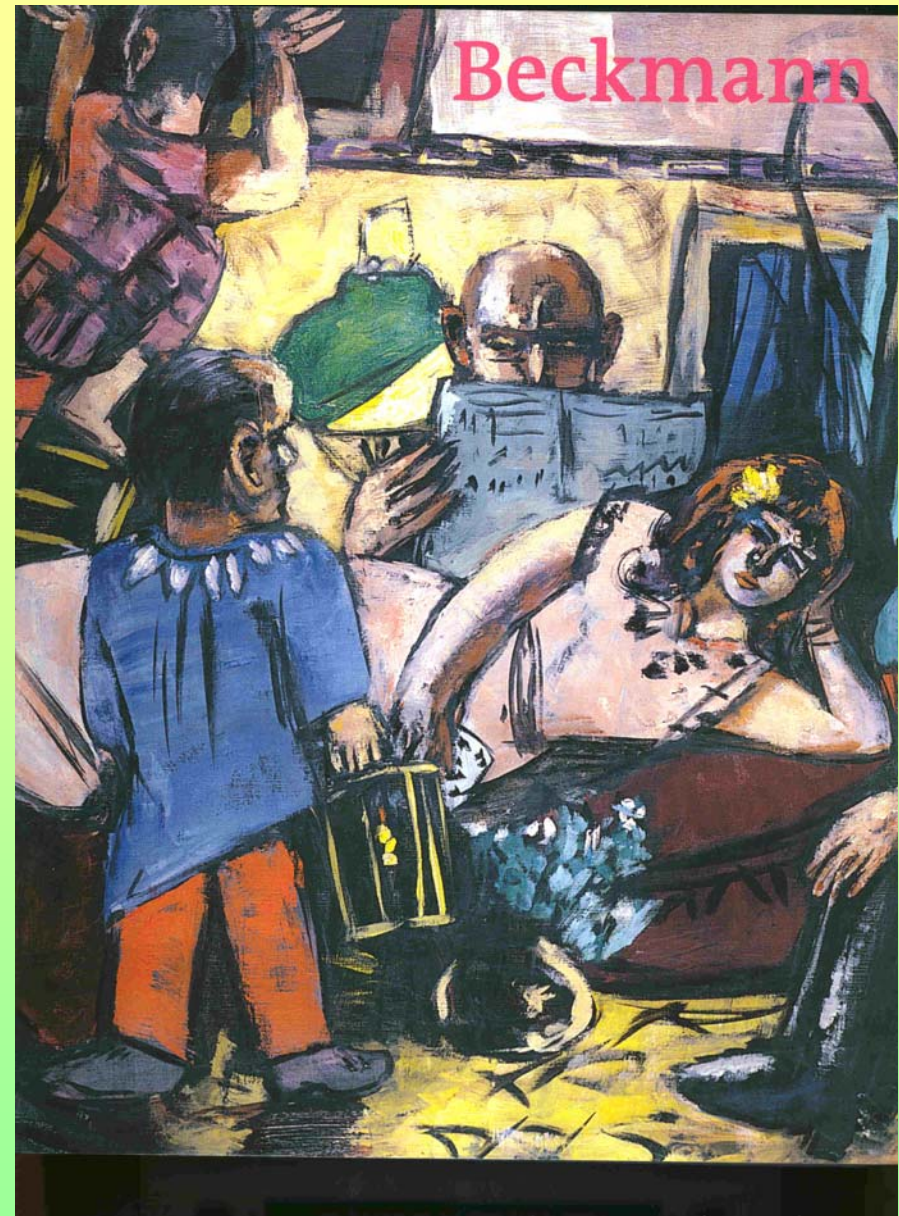
George Grosz, 1919



George Grosz, 1922



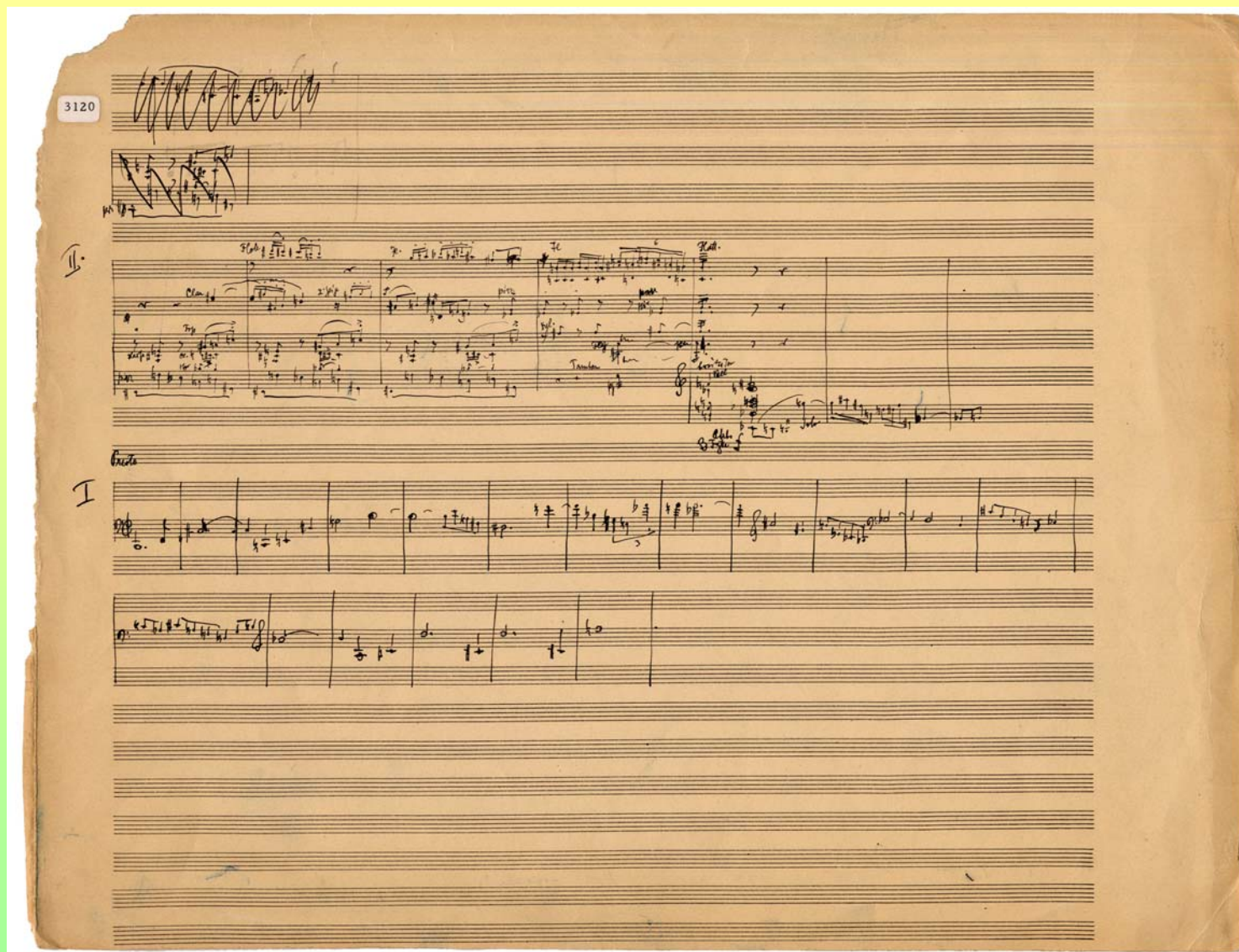
1922





**Die Frau ohne Schatten** (op. 65)  
Oper in drei Aufzügen.  
Text von Hugo von Hofmannsthal.  
Das Werk wurde am 10. Oktober 1919  
in Wien uraufgeführt

Richard Strauss, Gemälde von Max Liebermann, 1918



Arnold Schönberg:  
Die Jakobsleiter, about 1920



# 1920

Freud erstellt im Zusammenhang mit den Anschuldigungen an Julius Wagner Jauregg *Das Gutachten Über die elektrische Behandlung von Kriegsneurotikern*. Julius Wagner Jauregg war beschuldigt worden, Soldaten mit Elektroschocks mißhandelt zu haben. Freud kritisiert Wagner Jaureggs Methode, entlastet ihn aber vom Vorwurf, Patienten mit Absicht gequält zu haben.

Freuds Tochter Sophie Halberstadt stirbt in Hamburg an den Folgen einer Grippe.

Die englischsprachige psychoanalytische Zeitschrift *International Journal of Psycho-Analysis* wird gemeinsam mit Ernest Jones in Wien gegründet.





Schiele, Egon, \* 12. 6. 1890 Tulln (Niederösterreich), † 31. 10. 1918 Wien



Klimt, Gustav, \* 14. 7. 1862 Wien, † 6. 2. 1918



**Henri Poincaré's** father was Léon Poincaré and his mother was Eugénie Launois. Léon Poincaré's family produced other men of great distinction during Henri's lifetime. Raymond Poincaré, who was prime minister of France several times and president of the French Republic during World War I, was the elder son of Léon Poincaré's brother Antoine Poincaré. The second of Antoine Poincaré's sons, Lucien Poincaré, achieved high rank in university administration.



**Fernand Leger**  
*The City*, 1919



**Pablo Picasso: 1922**



# Igor Stravinsky

drawing: P. Picasso

**1920**

## **Pulcinella**

Ballet after Pergolesi for Dancer-  
Pantomimes, Singers and Chamber  
Orchestra

## **Concertino**

for String Quartet

## **Symphonies of Wind Instruments**

for 23 Winds

**1921**

## **Le cinq doigts (The Five Fingers)**

Eight Pieces for Piano

## **Suite Nr. 2**

for Chamber Orchestra



## Franz Kafka

"Das Schloss" (1926,  
entstanden 1922)



Alphonse Mucha:  
1921 Erfolgreiche  
Ausstellung  
im Brooklyn Museum

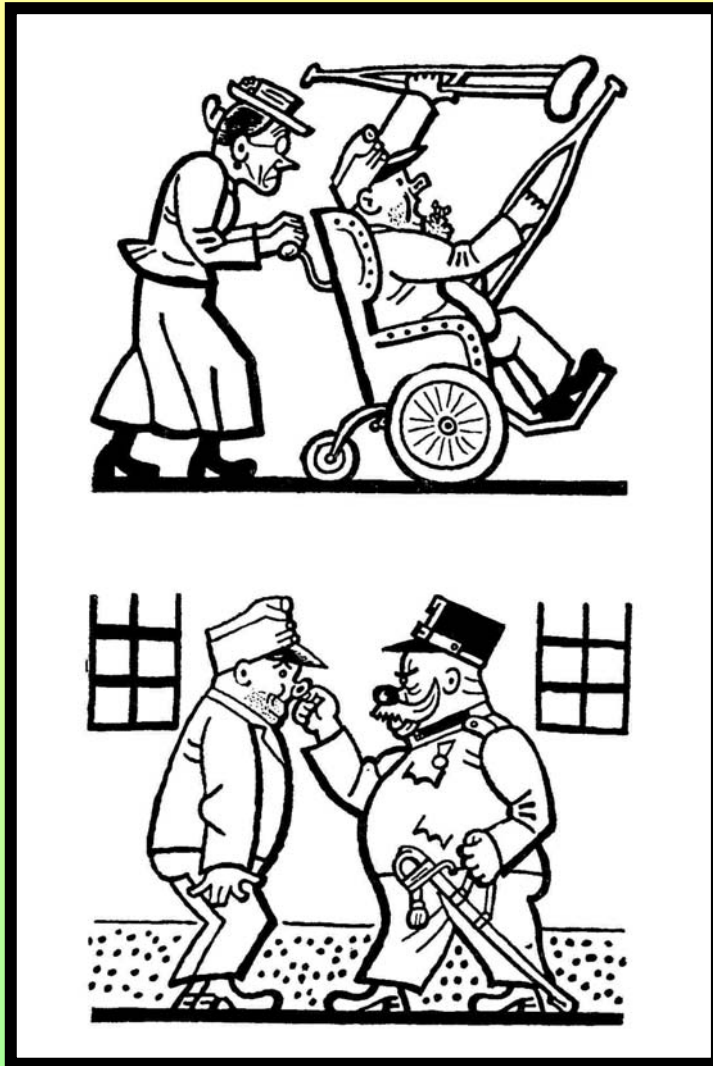


## Jaroslav Hašek

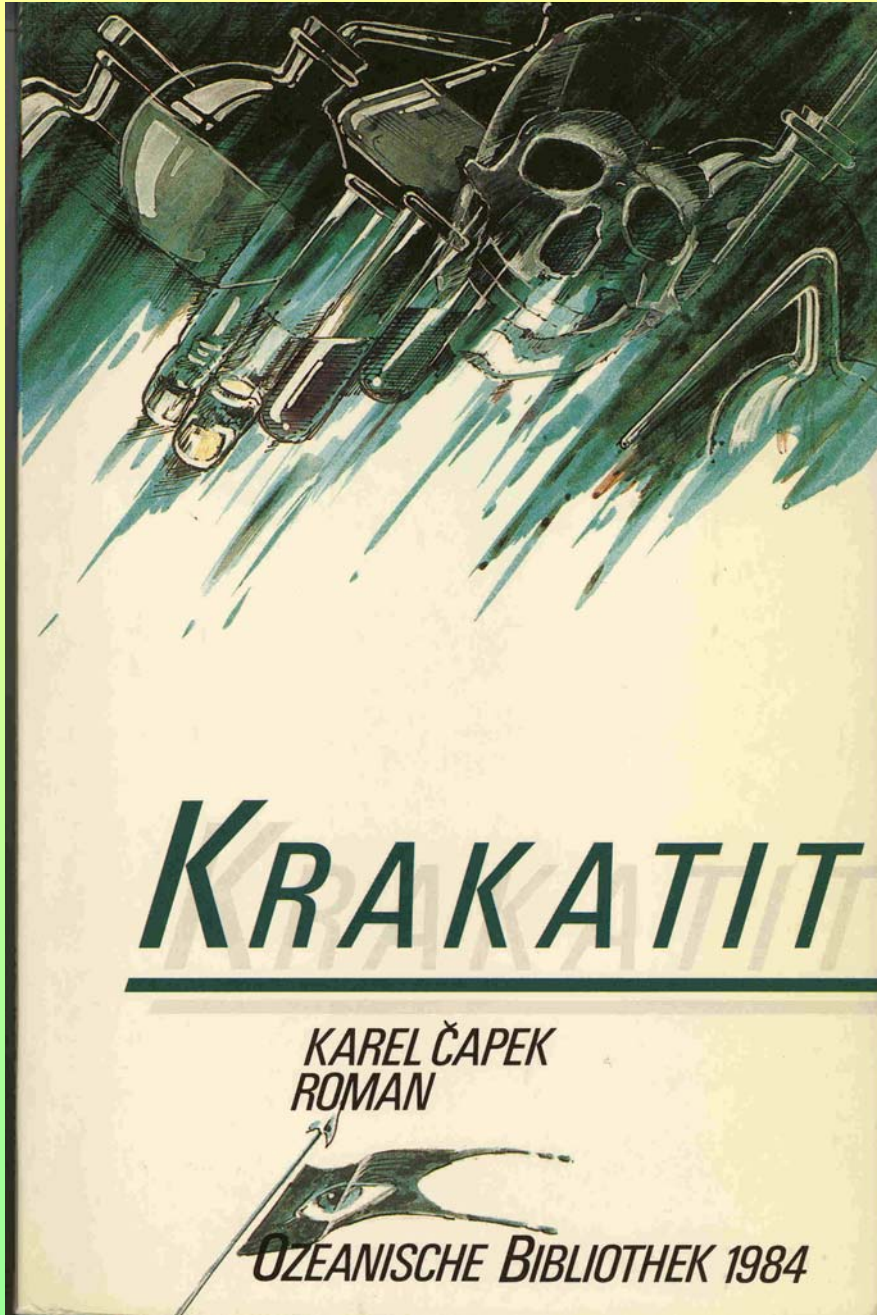
(\* [30. April 1883](#) in [Prag](#) in [Österreich-Ungarn](#);

† [3. Januar 1923](#) in [Lipnice nad Sázavou](#))





The first Sweijk story illustrated by Josef Lada appeared in 1922. Between 1924 and 1925 most of the by now famous illustrations by him were published in the sunday edition of „Ceske slava“.



This perhaps less well-known novel by Karel Capek was published in 1924

„Radioempfanganlage“, 1923  
Beginn des „professionellen“  
Rundfunks



Öffentlicher Verkehr



# A tentative theory of light quanta

L. De Broglie, *Phil. Mag.* 47, 446 (1924)

L. De Broglie, *J. de Physique* (November 1922)

# Preliminary I: Einstein's special theory of relativity

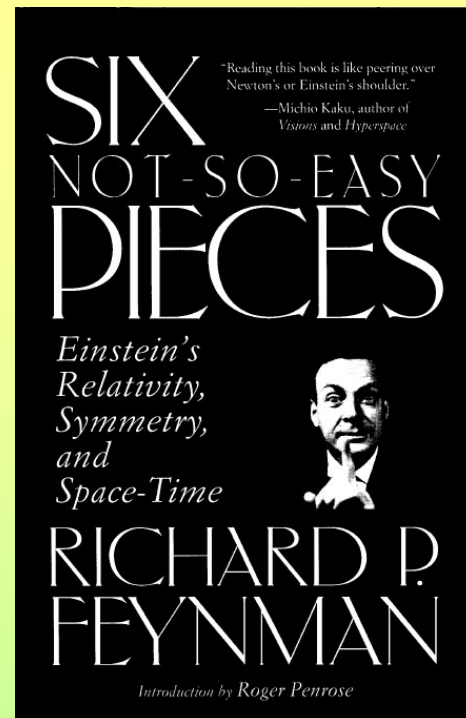
In 1905 Einstein surprised the physics community by stating that the tacit assumption of a constant mass  $m$  in Newton's second law,

$$F = \frac{d(mw)}{dt}, \quad (1)$$

$w$  being the velocity†, is void, and that  $m$  has to be corrected in the following way

$$m = \frac{m_0}{\sqrt{1 - w^2/c^2}}, \quad (2)$$

where  $m_0$ , the so-called rest mass, is the mass of a body that is not moving and  $c$  is the speed of light.



## Three

# THE SPECIAL THEORY OF RELATIVITY

### 3-1 *The principle of relativity*

◆ For over 200 years the equations of motion enunciated by Newton were believed to describe nature correctly, and the first time that an error in these laws was discovered, the way to correct it was also discovered. Both the error and its correction were discovered by Einstein in 1905.

Newton's Second Law, which we have expressed by the equation

$$F = d(mv)/dt,$$

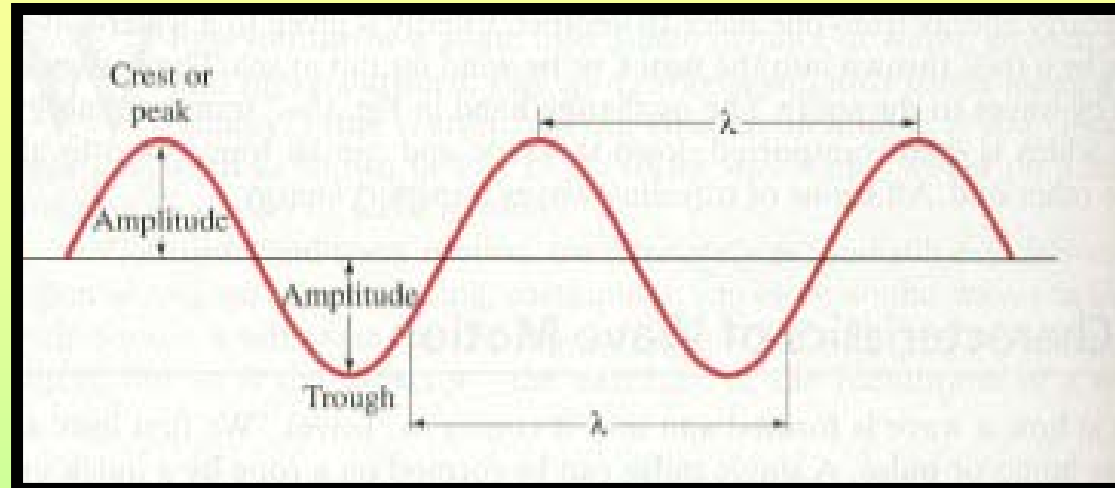
was stated with the tacit assumption that  $m$  is a constant, but we now know that this is not true, and that the mass of a body increases with velocity. In Einstein's corrected formula  $m$  has the value

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}, \quad (3.1)$$

where the "rest mass"  $m_0$  represents the mass of a body that is not moving and  $c$  is the speed of light, which is about  $3 \times 10^8$  km · sec<sup>-1</sup> or about 186,000 mi · sec<sup>-1</sup>.

For those who want to learn just enough about it so they can solve problems, that is all there is to the theory of relativity—it just changes Newton's laws by introducing a correction factor

# Preliminary II: Waves



The velocity (phase velocity)  $w$  [ $\text{cm s}^{-1}$ ] of propagating waves can simply be expressed in terms of the wavelength  $\lambda$  [ $\text{cm}$ ] and the frequency  $\nu$  [ $\text{s}^{-1}$ ]

$$w = \lambda \nu = \frac{\lambda}{T}, \quad (3)$$

where  $T$  is the period [s].

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I shall in the present paper assume the real existence of light quanta, and try to see how it would be possible to reconcile with it the strong experimental evidence on which was based the wave theory.

For the sake of simplicity, it is a very natural assumption to admit that all light quanta are identical

\* Communicated by R. H. Fowler, M.A.



## 2 A "very natural assumption"

Let us now first follow de Broglie's very natural assumption [3]: Let  $w$  be the velocity of a light quantum of frequency  $\nu$ , where  $w$  is **nearly close to Einstein's limiting velocity**  $c$ , and let us assume that all such **light quanta** are of the same mass  $m_0$ , then the energy  $W$  of one such quantum has to fulfill the relation

$$W = h\nu = \frac{m_0 c^2}{\sqrt{1 - \beta^2}} \quad , \quad \beta = \frac{w}{c} \quad . \quad (4)$$

This in essence is de Broglie's very first argument not mentioning, however, at this stage what  $h$  refers to, but this equation seems **natural**, since according to Bohr's theory atoms can only emit or adsorb radiant energy of frequency  $\nu$  by finite amounts equal to  $h\nu$ .

***bold: citations from de Broglie's paper***

Equation (4) is of course in complete agreement with Einstein, but, turned around,

$$\sqrt{1 - \beta^2} = \frac{m_0 c^2}{h\nu} \quad , \quad (5)$$

$$\beta = \sqrt{1 - \frac{m_0^2 c^4}{h^2 \nu^2}} \quad , \quad (6)$$

poses already a fundamental problem, namely of how to linearize a square root, i.e., a problem is addressed that was solved only years after by Dirac. However, since  $w$  is close to  $c$ , as de Broglie argues, he simply uses a binomial series, namely that for  $(1 - x)^{1/2}$ , valid for  $|x| \leq 1$ , and by truncating the series after the first term he arrives at the expression

$$\beta = \frac{w}{c} = \sqrt{1 - \frac{m_0^2 c^4}{h^2 \nu^2}} \simeq 1 - \frac{1}{2} \frac{m_0^2 c^4}{h^2 \nu^2} \sim 1 \quad , \quad (7)$$

from which he concludes that  $m_0$  has to be very small indeed, *at most of the order of*  $10^{-50}$  [g]. The possibility that the velocity of all light quanta might equal **Einstein's limiting velocity** he carefully excluded, since then of course according to his argument the mass of **light atoms** had to be zero,

$$\beta = 1, \forall \nu \rightarrow m_0 = 0 \quad . \quad (8)$$

### 3 Black radiation and "Planck's hypothesis"

As in de Broglie's *very natural assumption*  $c \sim w$ , Eq. (4) can approximately be written as


$$W = \frac{m_0 c^2}{\sqrt{1 - \beta^2}} \sim \frac{m_0 c w}{\sqrt{1 - \beta^2}} \quad , \quad (10)$$

which implies by considering only the corresponding units that  $W/c$  must be a momentum

$$G = \frac{W}{c} = \frac{m_0 w}{\sqrt{1 - \beta^2}} \quad , \quad (11)$$

namely the (modulus of the) momentum of one light quantum.

Now, if this indeed is a momentum clearly the question arises what kind of distribution law, i.e., what kind of statistical mechanics, applies? Is it a classical one? Is Maxwell's energy partition law the basis of further understanding? Does one need to consider a gas of light quanta? Clearly enough despite all attempts of Ernst Mach, Boltzmann statistics was already well accepted at de Broglie's time, but – as we know now - is only valid in the thermodynamical limit. At this point in his paper de Broglie recalls an earlier study [4] of his in which he showed that the phase space element  $d\Omega$  to be used in statistical mechanics has to be scaled down by exactly that constant that Planck had suggested, namely by the quantity  $h$ , which implicitly was already used in Eq. (4),

$$d\Omega = \frac{1}{h^3} dx dy dz dp_x dp_y dp_z \quad . \quad (12)$$


In that study he had arrived at the following density  $u_\nu$  of the radiant energy

$$u_\nu d\nu = \frac{8\pi h}{c^3} \nu^3 e^{-h\nu/kT} d\nu \quad . \quad (13)$$

With respect to the results in Eqs. (12) and (13), de Broglie adds the following comment: This was an encouraging result, but not quite complete. The assumption of finite elements of extension in phase space seemed to have a somewhat arbitrary and mysterious character. The result in Eq. (13) – as he states – was only possible by using Planck's hypotheses in Eq. (12).

At this stage we simply have to admire de Broglie's speculations, since by "simply guessing" the form of Eq. (12) Heisenberg's uncertainty principle was already anticipated and of course the remaining mystery about the type of statistics to be applied (... **I was obliged to suppose some kind of quanta aggregation ..**) was only cleared up by the - to us now quite familiar - concepts of quantum statistics, i.e., by the Bose-Einstein statistics in the case of photons (**light quanta, light atoms**). But, and this is important to stress, the famous factor  $h^{-3}$  seemed to be reasonable already in 1922 (!).

Of course now, more than eighty years after, we might think that his expression for the pressure  $p$  on the wall of a gas of light quanta in the context of Einstein's theory of relativity appears a bit far fetched, since in plucking the momentum  $G$  from Eq. (11) into the well-known expression for the pressure (isotropical distribution of velocities) de Broglie arrives proudly at the expression,

$$p = \frac{n}{6} \cdot (2Gc) = \frac{1}{3}nW \quad , \quad (14)$$

where  $n$  is the number of light quanta in a volume element, that differs by a factor two from the traditional (Maxwell-Boltzmann) formulation. However, the arguments leading to Eq. (14) illustrate perfectly the spell that Einstein's theory was exerting on the physics community in the early twenties and seemingly partially still does considering that in 2005 ("A hundred years of the special theory of relativity") the world-wide Year of Physics turned out to be a Year of Einstein.

## 4 Dynamics and Bohr's stability conditions

The third major type of argument de Broglie discusses is directed to the concept of time. From Eq. (4) follows immediately that the frequency  $\nu$  can be expressed in terms of the energy  $W$ ,

$$\nu = \frac{W}{h} = \frac{1}{h} \frac{m_0 c^2}{\sqrt{1 - \beta^2}} \quad , \quad \beta = \frac{w}{c} \quad , \quad (15)$$

which multiplied by  $\beta^2$  yields

$$\beta^2 \nu = \frac{w^2}{c^2} \nu = \frac{1}{h} \beta^2 \frac{m_0 c^2}{\sqrt{1 - \beta^2}} \quad . \quad (16)$$

In order to avoid the rather complicated arguments given by de Broglie, let us assume that on the left hand side  $\beta$  is about 1,

$$\frac{w^2}{c^2} \frac{w}{\lambda} \sim \frac{w}{\lambda} = \frac{1}{h} \frac{m_0 \beta^2 c^2}{\sqrt{1 - \beta^2}} \quad , \quad (17)$$

and, furthermore, let us recall that  $w$  is the phase velocity,

$$v = \frac{1}{T} = \frac{w}{\lambda} \quad ,$$

where  $\lambda$  is the wavelength and  $T$  is the period, see also Eq.(3). If  $ds$  denotes an element of the path of a wave to move from one crest to the next one, then

$$\int \frac{ds}{\lambda} = n \quad , \quad (18)$$

where  $n$  is the number of periods  $T$  [s], or the number of times this wave assumes the value of its amplitude. Integrating therefore the rhs of Eq.(17) over the time  $T$  has to yield the same result as the line integral on the lhs of this equation, i.e.,

$$\int \frac{ds}{\lambda} = \int_0^T \frac{1}{h} \frac{m_0 \beta^2 c^2}{\sqrt{1 - \beta^2}} dt = n \quad (19)$$

This now is as De Broglie says *an interesting explanation of Bohr's stability conditions*, which states that "the motion (of an electron) can only be stable if the phase wave is tuned with the length of the path". Eq. (19) is indeed astonishing, since – as already said – to the lhs there is a line integral and on the rhs an integral over the time, and also because de Broglie treats photons as if they would be one of Bohr's electrons. With Eq. (19) de Broglie opened the door to "wave mechanics" as it was called then, a door through which Schrödinger would be passing only two years later. In Schrödinger's first paper [5], which can be regarded as the actual starting point of quantum mechanics, he very honestly admits that all his thinking started from the witty ideas of de Broglie. Schrödinger gives complete reference to Eq. (19) as the origin of all his investigations by citing de Broglie's thesis [7], which in turn is the basis of the paper in the Philosophical Magazine.



E. Schrödinger, Ann. Phys. 79, 361 (1926)

"Vor allem möchte ich nicht unerwähnt lassen, daß ich die Anregung zu diesen Überlegungen in erster Linie den geistvollen Thesen des Hrn. Louis de Broglie verdanke und dem Nachdenken über die räumliche Verteilung von jener Phasenwellen, von denen er gezeigt hat, daß ihrer stets eine ganze Zahl, entlang der Bahn gemessen, auf jede Periode oder Quasiperiode des Elektrons fallen". ("I would like to stress here that primarily the witty theses of Mr. L. deBroglie lead me to these ideas and to thinking about the spacial distribution of such phase waves, where he proved that always an integer number, measured along the orbit, falls on each period or quasi period of the electron").

Rereading de Broglie's paper in the Philosophical Magazine makes clear that in 1924 quite a few aspects that nowadays seem to be completely familiar to us were totally unclear then, were an enigma, as De Broglie admits. It is indeed remarkable that he occasionally speaks of atoms of light when he talks about phonons. This in turn is a very old idea, namely that not only matter *per se* cannot be divided infinitely (Demokritos), but also that time can only be chopped down to indivisible units [8] ("Zeitatom"), which to some extent correspond to the "phase space volume factor"  $h^3$ , i.e., refer to a concept closely related to Heisenberg's uncertainty principle. de Broglie deals with photons and relates them to the orbits of an electron in Bohr's atomic model, since in 1924 a distinction between "Fermions" and "Bosons" did not exist nor was it thinkable! And in principle the main idea about this particular paper is to make use of Einstein's correcting factor  $\beta$  by equating cleverly this factor in the right place to unity.

There is of course one final observation to be made: in terms of the present politics of publishing scientific papers, de Broglie's contribution could never have been published because it contains essentially speculations only. However, one just as well can say, this paper proves that speculations are an essential part of physics, without them no new ideas and theories are born. And quantum mechanics has to be regarded as a true rupture in the history of physics, as a revolution in the philosophy of science, a revolution that desperately needed speculations and deviations beyond well-accepted ways of thinking.

their velocities are different. We shall then assume that the "mass at rest" of every light quantum has a given value  $m_0$ : since the atoms of light have velocities very nearly equal to the Einstein's limiting velocity  $c$ , they must have an extremely small mass (not infinitely small in a mathematical sense); the frequency of the corresponding radiation must be related to the whole energy of a quantum by the relation

$$h\nu = \frac{m_0 c^2}{\sqrt{1-\beta^2}}, \quad \left(\beta = \frac{v}{c}\right);$$

but, since  $1-\beta^2$  is very small, we can write

$$\beta = \frac{v}{c} = 1 - \frac{1}{2} \frac{m_0^2 c^4}{h^2 \nu^2}.$$

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*M. L. de Broglie on  $\alpha$* 

or also

$$\int_0^T \frac{m_0}{\sqrt{1-\beta^2}} (v_x^2 + v_y^2 + v_z^2) dt = \int_0^T \frac{m_0}{\sqrt{1-\beta^2}} \beta^2 c^2 dt = nh,$$

which is precisely the result obtained above.

*Philosophical Magazine Letters*,  
Vol. 86, No. 7, July 2006, 405–410



Taylor & Francis  
Taylor & Francis Group

**Revisiting Louis de Broglie's famous 1924 paper in the  
*Philosophical Magazine***

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