XXXV. *A Tentative Theory of Light Quanta.* *By* LOUIS DE BROGLIE *.

I. *The Light Quantum.*

THE experimental evidence accumulated in recent years seems to be quite conclusive in favour of the actual reality of light quanta. The photoelectric effect, which is the chief mechanism of energy exchange between radiation and matter, seems with increasing probability to be always governed by Einstein's photoelectric law. Experiments on the photographic actions, the recent results of A. H. Compton on the change in wave-length of scattered X-rays, would be very difficult to explain without using the notion of the light quantum. On the theoretical side Bohr's theory, which is supported by so many experimental proofs, is grounded on the postulate that atoms can only emit or absorb radiant energy of frequency ν by finite amounts equal to $h\nu$, and Einstein's theory of energy fluctuations in the black radiation leads us necessarily to the same ideas.

I shall in the present paper assume the real existence of light quanta, and try to see how it would be possible to reconcile with it the strong experimental evidence on which was based the wave theory.

For the sake of simplicity, it is a very natural assumption to admit that all light quanta are identical and that only

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their velocities are different. We shall then assume that the "mass at rest" of every light quantum has a given value m_0: since the atoms of light have velocities very nearly equal to the Einstein's limiting velocity c, they must have an extremely small mass (not infinitely small in a mathematical sense); the frequency of the corresponding radiation must be related to the whole energy of a quantum by the relation

$$h\nu = \frac{m_0 c^2}{\sqrt{1-\beta^2}}, \quad \left(\beta = \frac{v}{c}\right);$$

but, since $1-\beta^2$ is very small, we can write

$$\beta = \frac{v}{c} = 1 - \frac{1}{2}\frac{m_0{}^2 c^4}{h^2\nu^2}.$$

The light quanta would have velocities of slightly different values, but such that they cannot be discriminated from c by any experimental means. It then seems that m_0 should be at most of the order of 10^{-50} gr.

Naturally, the light quantum must have an internal binary symmetry corresponding to the symmetry of an electromagnetic wave and defined by some axis of polarization. We shall refer again later to this remark.

II. *The Black Radiation as a Gas of Light Quanta.*

Let us consider a gas made up by the light quanta we have described above. At a given temperature (not too near to the absolute zero) almost all these atoms of light would have velocities $v = \beta c$ very nearly equal to c. The whole energy of one of these atoms is

$$W = \frac{m_0 c^2}{\sqrt{1-\beta^2}},$$

and its momentum is

$$G = \frac{m_0 v}{\sqrt{1-\beta^2}};$$

so we have approximately

$$G = \frac{W}{c}.$$

The pressure of such a gas on the walls of the enclosure is easily seen to be

$$p = \frac{n}{6} \cdot 2Gc = \tfrac{1}{3} nW,$$

if n is the number of light quanta in an element of volume.

This expression is the same as the one given by the electromagnetic theory, whilst without using the Relativity formulæ we should have found a result twice as great as this.

Now the question arises, Can we use for the quanta gas Maxwell's energy partition law? In Einstein's dynamics, Liouville's theorem, on which is based all Statistical Dynamics, is still valid; we can then use for the elementary cell of extension-in-phase a value proportional to $dx\,dy\,dz\,dp\,dq\,dr$, if x, y, z are rectangular coordinates and p, q, r the corresponding momenta. In consequence of the canonical distribution law, the number of atoms whose representative points are in the element $dx\,dy\,dz\,dp\,dq\,dr$ must be proportional to

$$e^{-\frac{W}{kT}}dx\,dy\,dz\,dp\,dq\,dr = e^{-\frac{W}{T}}.\,4\pi G^2\,dG\,dv,$$

if dv is the element of volume and G the momentum. But, since $G=\frac{W}{c}$, this number is also given by

$$C^t \times e^{-\frac{W}{kT}}w^2\,dw\,dv.$$

Each quantum has a total energy $h\nu$; then the whole energy contained in the volume dv and carried by light quanta of energy $h\nu$ is

$$C^t e^{-\frac{h\nu}{kT}}\nu^3\,d\nu\,dv.$$

This is obviously Wien's limiting form of the radiation law. Two years ago * I was able to show that, by using the hypotheses made by Planck that the element of extension in phase was $\frac{1}{h_3}dx\,dy\,dz\,dp\,dq\,dr$, it was possible to find for the radiant energy density the value

$$u_\nu\,d\nu = \frac{8\pi h}{c^3}\nu^3 e^{-\frac{h\nu}{kT}}d\nu.$$

This was an encouraging result, but not quite complete. The assumption of finite elements of extension in phase seemed to have a somewhat arbitrary and mysterious character. Moreover, Wien's law is only a limiting form of the actual radiation law, and I was obliged to suppose some kind of quanta aggregation for explaining the other terms of the series.

It seems that these difficulties are now removed, but we shall first of all explain many other ideas; we shall later on return to the "black radiation" gas.

* See *Journal de Physique*, November 1922.

III. *An important Theorem on the Motion of Bodies.*

Let us consider a moving body whose "mass at rest" is m_0; it moves with regard to a given observer with velocity $v=\beta c$ $(\beta<1)$. In consequence of the principle of energy inertia, it must contain an internal energy equal to m_0c^2. Moreover, the quantum relation suggests the ascription of this internal energy to a periodical phenomenon whose frequency is $\nu_0=\frac{1}{h}m_0c^2$. For the fixed observer, the whole energy is $\frac{m_0c^2}{\sqrt{1-\beta^2}}$ and the corresponding frequency is $\nu=\frac{1}{h}\frac{m_0c^2}{\sqrt{1-\beta^2}}$.

But if the fixed observer is looking at the internal periodical phenomenon, he will see its frequency lowered and equal to $\nu_1=\nu_0\sqrt{1-\beta^2}$, that is to say this phenomenon seems for him to vary as $\sin 2\pi\nu_1 t$. The frequency ν_1 is widely different from the frequency ν; but they are related by an important theorem which gives us the physical interpretation of ν.

Let us suppose that, at time 0, the moving body coincides in space with a wave whose frequency ν has the value given above and which spreads with velocity $\frac{c}{\beta}=\frac{c^2}{v}$. This wave, however, cannot carry energy, according to Einstein's ideas,

Our theorem is the following:—"*If, at the beginning, the internal phenomenon of the moving body is in phase with the wave, this harmony of phase will always persist.*" In fact, at time t, the moving body is at a distance from the origin $x=vt$ and its internal phenomenon is proportional to $\sin 2\pi\nu_1\frac{x}{v}$; at the same place the wave is given by $\sin 2\pi\nu\left(t-\frac{\beta x}{c}\right)=\sin 2\pi\nu\, x\left(\frac{1}{v}-\frac{\beta}{c}\right)$. The two sines will be equal; the harmony of phase will again occur if the following condition is realized:

$$\nu_1=\nu(1-\beta^2),$$

a condition clearly satisfied by the definitions of ν and ν_1.

This important result is implicitly contained in Lorentz's time transformation. If τ is the local time of an observer carried along with the moving body, he will define the internal phenomenon by the function $\sin 2\pi\nu_0\tau$. According to Lorentz's transformation, the fixed observer must describe the same phenomenon by the function $\sin 2\pi\nu_0 \frac{1}{\sqrt{1-\beta^2}}\left(t-\frac{\beta x}{c}\right)$, which can be interpreted as the representation of a wave of frequency $\frac{\nu_0}{\sqrt{1-\beta^2}}$ spreading along the x axis with velocity $\frac{c}{\beta}$.

We are then inclined to admit that any moving body may be accompanied by a wave and that it is impossible to disjoin motion of body and propagation of wave.

This idea can also be expressed in another way. A group of waves whose frequencies are very nearly equal has a "group velocity" U, which has been studied by the late Lord Rayleigh, and which in the usual theory is the velocity of "energy propagation." This group velocity is linked with the "phase velocity" V by the relation

$$\frac{1}{\mathrm{U}} = \frac{d\left(\frac{\nu}{\mathrm{V}}\right)}{d\nu}.$$

If ν is equal to $\frac{1}{h}\frac{m_0c^2}{\sqrt{1-\beta^2}}$ and V to $\frac{c}{\beta}$, we find $\mathrm{U}=\beta c$—that is to say, "*The velocity of the moving body is the energy velocity of a group of waves having frequencies* $\nu=\frac{1}{h}\cdot\frac{m_0c^2}{\sqrt{1-\beta^2}}$ *and velocities* $\frac{c}{\beta}$ *corresponding to very slightly different values of* β."

IV. *Dynamics and Geometrical Optics.*

Trying to extend the former ideas to the case of variable velocity is a rather difficult but very suggestive problem. If in any medium a moving body describes a curved path, we say that there is a field of force; at each point the potential energy may be calculated, and the body when crossing this point has a velocity determined by the constant value of its whole energy. Now, it seems natural to suppose that the phase wave must have at any point a velocity and a frequency fixed by the value *which* β *would have if the body were there.* During its propagation the phase wave has a constant frequency ν and a constantly variable velocity V.

Perhaps a new electromagnetism will give us the laws of this complicated propagation, but it seems that we know beforehand the final result: "*The rays of the phase wave are identical with the paths which are dynamically possible.*" In fact, the paths of the rays can be computed as in a medium of variable dispersion by means of Fermat's principle, which may be written here (λ wave-length, ds element of path):

$$\delta \int \frac{ds}{\lambda} = \delta \int \frac{\nu\, ds}{V} = \delta \int \frac{m_0 \beta c}{\sqrt{1-\beta^2}} ds = 0.$$

The principle of least action in its Maupertuisian form gives the dynamical paths by the equation

$$\delta \int m_0 c^2 \left(\frac{1}{\sqrt{1-\beta^2}} - \sqrt{1-\beta^2} \right) dt$$
$$= \delta \int \frac{m_0 \beta^2 c^2}{\sqrt{1-\beta^2}} dt = \delta \int \frac{m_0 \beta c}{\sqrt{1-\beta^2}} ds = 0,$$

a result which justifies the above statement.

It is now so simple a matter to show that the theorem of phase harmony is always valid that it seems not necessary to develop the proof.

The present theory suggests an interesting explanation of Bohr's stability conditions. At time 0 the electron is in a point A of its trajectory. The phase wave starting at this instant from A will describe all the path and meet again the electron in A′. It seems quite necessary that the phase wave shall find the electron in phase with itself. This is to say: "The motion can only be stable if the phase wave is tuned with the length of the path." The tune relation is then:

$$\int \frac{ds}{\lambda} = \int_0^T \frac{m_0 \beta^2 c^2}{h\sqrt{1-\beta^2}} dt = n$$

(n whole number; T revolution period).

Now, we can write the stability condition of the quantum theory in a general form given by Einstein which degenerates into the manifold Sommerfeld's conditions for quasi-periodical cases in consequence of the infinite number of the pseudo-periods. Let us call $p_x\, p_y\, p_z$ the momenta; then Einstein's general condition is

$$\int (p_x\, dx + p_y\, dy + p_z\, dz) = nh \quad (n \text{ whole number}),$$

or also

$$\int_0^T \frac{m_0}{\sqrt{1-\beta^2}}(v_x^2+v_y^2+v_z^2)\,dt = \int_0^T \frac{m_0}{\sqrt{1-\beta^2}}\,\beta^2c^2dt = nh,$$

which is precisely the result obtained above.

V. *The Propagation of Light Quanta and the Coherence Problem.*

We will now make use of our results for studying the propagation of free light quanta whose velocities are always slightly lower than c. We can say: "The atom of light whose whole energy is equal to $h\nu$ is the seat of an internal periodical phenomenon which, for a fixed observer, has at each point of space the same phase as a wave spreading in the same direction with velocity very nearly equal to c (very little greater)." The light quantum is in some manner a part of the wave, but for explaining interferences and other phenomena of the wave optics it is necessary to see how several light quanta can be parts of the *same* wave. This is the coherence problem.

In the light quanta theory, it seems necessary to make the following hypothesis: "When a phase wave crosses an excited atom, this atom has a certain probability of emitting a light quantum determined at each instant by the intensity of the wave." Perhaps this hypothesis will seem arbitary, but I think that any theory of coherence must adopt some postulate of this kind.

The emissions of γ-rays by radioactive substances are known to be quite independent, but this cannot be considered as an objection against our view because the "mean life" of any known radioactive atom is always so much greater than the period of the γ-rays.

Thus when an atom emits a light quantum, a spherical phase wave is simultaneously emitted and, crossing over the neighbouring atoms of the point source, will excite other emissions. The non-material phase wave will carry many little drops of energy which slide slowly upon it and whose internal phenomena are coherent.

VI. *Diffraction by a Screen Edge and the Inertia Principle.*

The corpuscular theory of light meets here a great difficulty. It is known since Newton that the light rays passing within a short distance of a screen edge are no longer straight but that some penetrate into the geometrical shadow. Newton ascribed this deviation to the action of some forces which would be exerted by the edge upon the light corpuscle. It seems to me that this phenomenon is perhaps worthy of a more general explanation. Since an intimate connexion seems to exist between motion of bodies and propagation of waves, and since the rays of the phase wave may now be considered as the paths (the possible paths) of the energy quanta, we are inclined to give up the inertia principle and to say: "A moving body must always follow the same ray of its phase wave." In the continuous spreading of the wave, the form of the surfaces of equal phase will change continuously and the body will always follow the common perpendicular to two infinitely near surfaces.

When Fermat's principle is no longer valid for computing the ray path, the principle of least action is no longer valid for computing the body path. I think that these ideas may be considered as a kind of synthesis of optics and dynamics.

We must still specify some points. The ray which now assumes in our ideas an important physical significance may be defined as above by the *continuous* spreading of a small part of the phase wave: it cannot be defined at each point by the geometrical sum for all the waves of the vector which is called in electromagnetic theory "radiant or Poynting's vector." Let us consider a sort of Wiener's experiment. We send a train of plane waves on a perfectly reflecting plane mirror in the normal direction; standing waves are set up; the reflecting mirror is a nodal plane for the electric vector, the plane at a distance $\frac{1}{4}\lambda$ from the mirror is a nodal plane for the magnetic vector, the plane at a distance $\frac{1}{2}\lambda$ of the mirror is again a nodal plane for the electric vector, and so on. In each nodal plane the radiant vector is null. May we say that these planes are not crossed by any energy? Evidently not, we can only say that the interference states in these planes are always the same. In every interference case we should find similar intricacies. In the wave theory, the propagation of energy has a somewhat fictitious character, but, in return, the exact calculation of the interference fringes is easily made; we shall try in the next paragraph to see why it is so.

VII. *A new Explanation of Interference Fringes.*

Consider how we can detect the presence of light at a point in space—by direct perception of the scattered light, by photographic tests, by calorific effects, and perhaps by some other means. It seems that all these means can, in fact, be reduced to photoelectric actions and scattering. Now, when a light quantum crosses a material atom, it has a certain probability of being absorbed or scattered, which can depend on external agencies. If then a theory succeeds in determining these probabilities without taking account of the actual motion of energy, it may be able to predict correctly the average reaction between radiation and matter at each place. Following the electromagnetic theory (and Bohr's correspondence principle is consistent with this view) I have been inclined to suppose that, for a material atom, the probability of absorbing or scattering a light quantum is determined by the geometrical sum of one of the defining vectors of the phase waves crossing upon it. The last hypothesis is, in fact, quite analogous to that which is admitted by electromagnetic theory when it links intensity of disclosable light with the intensity of the resultant electric vector. Thus, in the Wiener's experiment, the photographic action only occurs in the nodal planes of the electric vector; according to the electromagnetic theory, the luminous magnetic energy is not disclosable.

Let us now consider Young's interference experiment. Some atoms of light pass through the holes and diffract along the ray of the neighbouring part of their phase waves. In the space behind the wall, their capacity of photoelectric action will vary from point to point according to the interference state of the two phase waves which have crossed the two holes. We shall then see interference fringes, however small may be the number of diffracted quanta, however feeble may be the incident light intensity. The light quanta do cross all the dark and bright fringes; only their ability to act on matter is constantly changing. This kind of explanation, which seems to remove at the same time the objections against light quanta and against the energy propagation through dark fringes, may be generalized for all interference and diffraction phenomena.

VIII. *The Quanta and the Dynamical Theory of Gases.*

For the sake of calculating the entropy constants and the so-called "chemical constants," Planck and Nernst have been obliged to introduce the quantum idea into the theory of gases. As explained above, Planck chooses an element of extension in phase equal to

$$\frac{1}{h^3} dx\, dy\, dz\, dp\, dq\, dr \quad \text{or} \quad \frac{4\pi}{h^3} m_0^{3/2} \sqrt{2w}\, dw\, dx\, dy\, dz.$$

We shall now try to justify this assumption.

Each atom of velocity βc may be considered as linked with a group of waves whose phase velocity is $V=\frac{c}{\beta}$, frequency $\frac{1}{h}\frac{m_0c^2}{\sqrt{1-\beta^2}}$, and group velocity $U=\beta c$. The state of the gas can only be stable if the waves corresponding to all the atoms make up a system of standing waves. Using a well-known method given by Jeans, we find for the number of waves per unit of volume whose frequencies are included in the interval $\nu, \nu+d\nu$ * :

$$n_\nu d\nu = \frac{4\pi}{UV^2}\nu^2\,d\nu = \frac{4\pi}{c^3}\beta\nu^2\,d\nu.$$

If w is the kinetic energy of an atom and ν the corresponding frequency, then :

$$h\nu = \frac{m_0c^2}{\sqrt{1-\beta^2}} = w + m_0c^2 = m_0c^2(1+\alpha),$$

where $\alpha = \frac{w}{m_0c^2}$.

It is now very easy to find that $n_\nu d\nu$ is given by the equation :

$$n_\nu d\nu = \frac{4\pi}{h^3}m_0^2c(1+\alpha)\sqrt{\alpha(2+\alpha)}\,dw.$$

Each phase wave can carry with it one, two, or more atoms, so that, according to the canonical law, the number of atoms whose energy is $h.\nu$, will be proportional to :

$$\frac{4\pi}{h^3}m_0^2c(1+\alpha)\sqrt{\alpha(2+\alpha)}\,dw\,dx\,dy\,dz\sum_1^\infty e^{-\frac{nh\nu}{kT}}.$$

Let us first consider a material gas whose atoms have relatively great mass and relatively small velocities. We can then neglect all the terms of the series except the first, and we can also put $1+\alpha=1$. The number of atoms whose kinetic energy is w will be, neglecting a constant factor,

$$\frac{4\pi}{h^3}m_0^{3/2}\sqrt{2w}\,dw\,dx\,dy\,dz\,e^{-\frac{w}{kT}},$$

a result which justifies Planck's method and leads to the usual form of Maxwell's law.

In the case of the light quanta gas α is always great, and, moreover, we must use all the series. In consequence of the

* Léon Brillouin, *Théorie des Quanta*, p. 38. Paris: A. Blanchard.

internal binary symmetry of the light quantum, we must introduce a factor 2, and we find that the radiant energy density is proportional to :

$$\frac{8\pi}{h^3c^3}w^3\sum_1^\infty e^{-\frac{nh\nu}{kT}}dw = \frac{8\pi h}{c^2}\cdot\frac{\nu^3}{e^{\frac{h\nu}{kT}}-1}d\nu.$$

A method developed in the *Journal de Physique*, of November 1922, shows that the proportionality factor is unity, so that we obtain the actual radiation law.

IX. *Open Questions.*

The conceptions stated in this paper, if they are received, will necessitate a wide modification of the electromagnetic theory. The so-called "electric and magnetic energies" must be only a kind of average value, all the real energy of the fields being probably of a corpuscular fine-grained structure. The building up of a new electromagnetism seems to be a very difficult task, but we have one directive idea : according to the correspondence principle and to the above statements, the defining vectors of the old electromagnetic theory would give the probability of the reaction between matter and the fine-grained energy.

The new electromagnetism will give the solution of many problems. The laws of propagation of waves given by Maxwell's theory will probably be valid for the energyless light phase waves, and the scattering of the radiant energy will be explained by the resulting curvature of the rays (viz., light quanta paths). There seems to be a great analogy between scattering of radiation and scattering of corpuscles ; lowering of particles' velocities by crossing through a screen may also be similar to the lowering of X-ray frequencies by scattering, which recently has been computed and experimentally checked by A. H. Compton.

Explaining optical dispersion will be more difficult. The classical theories (including the electron theory) gives only an average view of this phenomenon, which is produced by complex elementary reactions between radiation and atoms ; we shall certainly be obliged here also to distinguish accurately the real motion of energy from the propagation of the resulting interference state. The kind of "resonance" shown by the variations of refractive index seems no longer to be irreconcilable with discontinuity of light.

Many other questions remain open : What is the mechanism of Bragg's absorption? What occurs when an atom passes from a stable state to another, and how does it eject a single quantum? How may we introduce the granular structure of energy into our conceptions of elastic waves and into Debye's theory of specific heats?

Finally, we must remark that the quantum relation remains still a kind of postulate defining the constant h whose actual significance is not at all cleared up; but it seems that the quantum enigma is now reduced to this unique point.

Summary.

In the present paper it is assumed that the light is essentially made up of light quanta, all having the same extraordinarily small mass. It is shown mathematically that the Lorentz-Einstein transformation joined with the quantum relation leads us necessarily to associate motion of body and propagation of wave, and that this idea gives a physical interpretation of Bohr's analytical stability conditions. Diffraction seems to be consistent with an extension of the Newtonian Dynamics. It is then possible to save both the corpuscular and the undulatory characters of light, and, by means of hypotheses suggested by the electromagnetic theory and the correspondence principle, to give a plausible explanation of coherence and interference fringes. Finally, it is shown why quanta must take a part in the dynamical theory of gases and how Planck's law is the limiting form of Maxwell's law for a light quanta gas.

Many of these ideas may be criticized and perhaps reformed, but it seems that now little doubt should remain of the real existence of light quanta. Moreover, if our opinions are received, as they are grounded on the relativity of time, all the enormous experimental evidence of the "quantum" will turn in favour of Einstein's conceptions.

1 October, 1923.

Note.—Since I have written this paper, I have been able to give to the results contained in the fourth section a somewhat different, but much more general form.

The principle of least action for a material point can be expressed in the space-time notation by the equation:—

$$\delta \int \sum_1^4 J_i \, dx^i = 0,$$

if the J_i are the covariant components of a four-dimensional vector whose time component is the energy of the point divided by c and space components are the components of its momentum.

Similarly, in studying the propagation of waves, we have to write :—

$$\delta \int \sum_{1}^{4} O_i \, dx^i = 0,$$

if the O_i are the covariant components of a four-dimensional vector whose time component is the frequency divided by c and space components the components of a vector drawn along the ray and equal to $\frac{\nu}{V} = \frac{1}{\lambda}$ (V phase velocity). Now, the quantum relation says that $J_4 = h\,O_4$. More generally, I suggest putting $\overrightarrow{J} = h\,\overrightarrow{O}$. From this statement, the identity of the two principles of Fermat and Maupertius follows immediately, and it is possible to deduce rigorously the velocity of the phase wave in any electromagnetic field.